Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Administration Manual for Supervising Teachers

January 2002

Manitoba
Education,
Training
and Youth

Preface

This *Administration Manual for Supervising Teachers* is distributed to teachers and administrators, and must be read prior to administering the standards test. The rules and guidelines in this manual must be strictly observed to ensure the consistency of standards test administration.

Please use this manual in conjunction with the registration package, *Policies and Procedures for Standards Tests* handbook and *Information Bulletin* to gain a full understanding of the protocols and procedures associated with this test. This manual contains general information about the test, step-by-step instructions for the test administration, and policies regarding absenteeism, lateness, violation of rules, etc.

If you encounter exceptional circumstances not addressed in this manual, or if you have further inquiries, contact Beatrice Davidson, Administrative Officer, at 945-8543, or toll-free at 1 800 282-8069, extension 8543.

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General Information

Manitoba Education, Training and Youth develops all standards tests in collaboration with teachers from across the province. This standards test is based on the *Senior 4 Mathematics Manitoba Curriculum Framework of Outcomes and Senior 4 Standards (2000).*

The standards test is locally marked and counts for 30% of each student's final grade in the course. Marking instructions are provided by the department immediately following test administration. Student results on the test will be released to students according to local policy on the date designated by local jurisdictions.

All students enrolled in Senior 4 Pre-Calculus Mathematics (40S), except those who have been exempted, must write the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* on January 22, 2002. Please note that all students repeating the Senior 4 Pre-Calculus Mathematics (40S) course(s) must write the test, even those who may have previously passed the provincial exam.

Students are expected to write the standards test as scheduled near the end of the semester in which they take the course (i.e., January for first-semester courses, May/June for second-semester and full-year courses).

All schools should receive the test materials by January 15, 2002. Principals of public schools receive test materials from their division/district offices. Principals of independent and band-operated schools will receive test materials directly from the Assessment and Evaluation Branch.

The number of test booklets sent to each school is based on the October 2001 registration information provided by the school. Please advise Beatrice Davidson, Administrative Officer, at 945-8543, or toll-free at 1 800 282-8069, extension 8543, immediately if the number of students registered to write the test has increased since registration.

School principals will receive *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* marking packages by January 22, 2002. The marking packages are to be stored in a secure location until 12:30 p.m. the day of the test.

In the context of marking standards tests at the local level, Manitoba Education, Training and Youth will:

- train representatives from local jurisdictions to act as local marking co-ordinators;
- collect and scan all marker sheets from jurisdictions to provide them with test result summaries;
- centrally mark a random sample of locally-marked test booklets to provide feedback to jurisdictions on their local marking;
- support provincially-funded local jurisdictions by providing $10.00 for each locally-marked test administered in accordance with the test schedule noted on the next page; and
- be responsible for preparing and distributing standards tests and accompanying materials, and for scanning test results once they have been locally marked.

Features of the test

The *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* consists of two parts:

- Part 1—multiple-choice and long-answer questions where a graphing or scientific calculator may be used; and
- Part 2—short-answer and long-answer questions where a calculator may not be used.

Test schedule

The following table summarizes the schedule for the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test*:

Date	Session	Preparation Time	Activity	Time Required for Test
Tuesday, January 22, 2002	1	15 minutes preparation and instructions	Part 1	120 min.
			Part 2	60 min.

The test must be written between 8:30 a.m. and 12:30 p.m. on Tuesday, January 22, 2002. Additional writing time cannot be permitted for either part. This also applies to those who may arrive late.

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Tuesday, January 22, 2002	1	15 minutes preparation and instructions	Part 1	120 min.
			Part 2	60 min.

The test must be written between 8:30 a.m. and 12:30 p.m. on Tuesday, January 22, 2002. Additional writing time cannot be permitted for either part. This also applies to those who may arrive late.

Policies for Standards Tests

Principals and supervising teachers should be aware that *Policies and Procedures for Standards Tests* handbook has been distributed to all schools and is available on-line at http://www.edu.gov.mb.ca/metks4/curricul/assess/index.html. This handbook outlines current policies and procedures related to the administration of standards tests and reflects the principles and guidelines contained in *Principles for Fair Student Assessment Practices for Education in Canada (1993).*

Adaptations

Schools are asked to identify and submit requests for test adaptations **at the time of student registration**. These requests should be submitted on the *Request for Adaptations Form* included with the registration package. Following departmental review of the adaptation requests received, schools will be advised of the department's decision regarding these requests. (*Policies and Procedures for Standards Tests,* Sections 4.0, 4.1 and 4.2)

Exemptions

Decisions for student exemptions that fall within the guidelines set out in *Policies and Procedures for Standards Tests* (Sections 5.0, 5.1 and 5.2) are made at the school level and do not require departmental approval. Schools are required, however, to report school-based exemptions on the *Student Exemption Report* included with the registration package. If additional student exemptions are needed after the registration deadline date, an additional *Student Exemption Report* should be submitted as soon as possible.

For exemptions that fall outside the parameters of policy and procedure guidelines, requests should be made **at the time of student registration**. Please submit requests in writing to Ernest Molgat, Director, Assessment and Evaluation Branch, 71–1567 Dublin Avenue, Winnipeg MB R3E 3J5.

Re-read provisions

As the reporting of individual student scores on locally marked standards tests is the responsibility of each local jurisdiction, the re-read policy for these tests is determined locally. Should a re-read be requested for a test booklet that has been sent to the department as a part of the feedback process, the department will return a photocopy of the test booklet upon request. Please contact Beatrice Davidson, Administrative Officer, at 945-8543, or toll-free at 1 800 282-8069, extension 8543.

Return of tests to students

Local jurisdictions decide whether to keep the test booklets or return them to students. The department will return the random sample of test booklets to schools within three months of marking.

Test confidentiality

Test materials must be kept secure and sealed until administration.

Test booklets must not be reproduced nor information from them communicated in any way until test administration is completed.

Student materials

Students may use the following materials during the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test*:

- HB lead pencil;
- calculator (scientific or graphing – Part 1 only);
- eraser; and
- ruler.

Students are not permitted to exchange materials during the test.

The following materials must **not** be used during the test:

- notes;
- textbooks; or
- dictionaries.

Students leaving the room

As the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* consists of two parts, students are expected to remain in the room for the allotted time for Part 1 and until they have completed Part 2 of the test.

However, any student who must leave and re-enter the room should be accompanied by a replacement supervisor. This practice ensures that the supervising teacher can remain in the test room with the students at all times.

Late arrivals

To avoid the opportunity for students to relay information during the test, students may not enter the test room after the first 60 minutes of writing.

Student absences

Standards tests cannot be written at a later date and reported as a standards test result. Therefore, schools have the responsibility to provide a final mark to absent students. The final mark for these students must be identified as a school-based assessment and evaluation practice only. (See *Policies and Procedures for Standards Tests*, Sections 6.0, 6.1 and 6.2, for additional information.)

Responding to student questions

Avoid making any directing/guiding comments during the test, other than those required for distribution of materials and instructions to students.

No explanations, translations, rephrasing, clarifying statements or comments of a similar nature may be provided about the test content. Classroom teachers can, however, provide clarification regarding test -taking rules or procedures.

If a student asks a question related to test content, a suggested response is, "I'm sorry. I can't answer your question. Do your best with it."

Suspected errors in printing or wording should not be communicated to students unless specific instructions are received from the Assessment and Evaluation Branch.

Violation of rules

The Test Rules are read aloud to students by the supervising teacher immediately before the test. (See Appendix 1, p. 17.)

Document any violation of rules on the *Supervising Teacher Report*. This documentation should include:

- the test booklet number; and
- a description of the circumstances, including available evidence.

The student may be permitted to complete the test.

The consequences of any violation of rules is a local decision.

Defective materials

Students should be assured that they will not be penalized for any obvious error in printing or wording. Local marking co-ordinators will be advised of any known errors before the marking session. Return the unused damaged booklet(s) with the reports and forms being sent to the department immediately after the test session.

On test administration date

If a damaged or imperfectly printed test booklet is discovered **at the start** of the test session:

- replace the defective test booklet with a new test booklet; and
- print "DAMAGED" on the test booklet cover and on the *Attendance Form*.

During test administration

If a damaged or imperfectly printed test booklet is discovered **during** the writing of the test:

- give the student a new test booklet (the student keeps the damaged test booklet until the end of the session);
- instruct the student to continue responding to test items in the new test booklet;
- collect both test booklets at the end of the test session;
- cross out the existing booklet number on the undamaged test booklet and change it to the original booklet number so that **all student responses are recorded under the original booklet number**;
- indicate on the *Attendance Form* that two booklets have been used for one student (see Appendix 3, p. 19);
- record the defective test booklet number on the *Supervising Teacher Report*; and
- ensure both booklets are kept together by bundling with an elastic band.

Responsibilities of School Principals

It is the responsibility of the principals to ensure that the tests are kept confidential until test administration is completed.

Other responsibilities are listed below.

If tests are not in your school by January 15, 2002:

- public school principals should contact their superintendent;
- principals of independent and band-operated schools should contact Beatrice Davidson, Administrative Officer, at 945-8543, or toll-free at 1 800 282-8069, extension 8543.

Use the *Packing List* to confirm that all materials have been received. Count the shrink-wrapped test booklets to ensure that there are enough booklets for the number of students writing the test. Keep the shrink-wrapped test packages unopened until the morning of the test. If additional test materials are needed, immediately contact Beatrice Davidson at the number noted above.

Store test materials in a secure area in the school.

Provide supervising teachers with a listing of the Manitoba Education, Training and Youth (MET) numbers for the students registered to write the test. Call the Professional Certification and Student Records Unit at 1 800 667-2378 to obtain MET numbers for new students.

Make arrangements to meet the requirements of the adaptations approved for your students (see Adaptations, p. 3 and *Policies and Procedures for Standards Tests*, Sections 4.0, 4.1 and 4.2).

Co-ordinate test scheduling and space requirements.

Minimize the frequency of school announcements during the test session.

Ensure that a replacement supervisor is available to accompany students who must temporarily leave the room.

Provide the test materials to the supervising teacher on the morning of the test.

Advise the Assessment and Evaluation Branch of school closures or suspended school bus transportation services that occur on the test date(s) as a result of unusual or extenuating emergency circumstances or extreme inclement weather conditions. Please contact Wenda Dickens, Test Development Co-ordinator, at 945-6774 or toll-free at 1 800 282-8069, extension 6774.

Complete the *Principal Feedback Form* before the end of the test session.

Responsibilities of Supervising Teachers

Supervising teachers administer the test using the step-by-step procedures outlined in the following section ("Checklist of Procedures") in this manual.

Points to Remember

- Remain in the classroom throughout the entire test session and engage in work solely related to the duties of supervising the test.
- A replacement supervisor must be available to assist you.
- Store all documents not in use in a secure area in the school.
- Ensure that once a booklet number has been assigned to a student, all materials distributed to that student have the same booklet number.
- Students **may** review or complete work from Part 1 during the Part 2 portion of the session but they **may not** use their calculator during this time.

Checklist of Procedures

	Advance Preparations
	1. Check with your principal to ensure that: • appropriate scheduling and space allocation for the test has occurred. • the test materials itemized on the *Packing List* have been inventoried. • a replacement supervisor is available to accompany students who must temporarily leave the room during the test. • arrangements have been made to accommodate the adaptations approved for students. • a list of MET numbers for students writing the test is available.
	2. Ensure that all subject-related materials have been removed from the walls and boards.
	3. Notify students in advance of the: • time, location and duration of the test session. • materials needed to write the test. • *Test Rules* (see Appendix 1, p. 17). • requirement to remain in the test session for the allotted time for Part 1 and until completion of Part 2.
	4. Ensure that you have extra supplies and that there is a functioning pencil sharpener and stapler in the room. Students must not exchange materials during the test.
	5. If there is more than one teacher supervising the test, photocopy the *Supervising Teacher Report* and *Supervising Teacher Feedback Form*.
	6. Review the sample forms in the Appendices.
	Prior to test session
	1. Ensure that your materials are available:
	• *Packing List*;
	• *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* booklets – Part 1 and Part 2;
	• *Attendance Forms* (enclosed with shrink-wrapped test booklets);
	• *Student Registration Update*;
	• extra copies of test booklets (four extras);
	• *Demographic Data Forms*;
	• *Answer/Scoring Sheets*;
	• *Supervising Teacher Report*;
	• *Supervising Teacher Feedback Form*;
	• *Principal Feedback Form* (this must be given to the principal and collected before returning materials);
	• *Administration Manual for Supervising Teachers* (this document); and
	• postage-paid envelope for return of materials to the Assessment and Evaluation Branch.

	2. Count the number of *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* booklets, Part 1 and 2. The number of test booklets should equal the number requested by the school plus the additional booklets. The number of booklets should also be identical to the number on the *Packing List*. If you need to split a bundle of booklets between classrooms, make a photocopy of the *Attendance Form* enclosed with the bundle. **On your copy**, cross out the test booklet numbers that are not used in your classroom.
	3. Arrange the desks so they are spaced as far apart as possible.
	4. Ensure that students exempted from writing the test are not present.
	Beginning the test session
	1. Distribute:
	• *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* – Part 1; Part 2 will be distributed in two hours.
	• *Demographic Data Forms*.
	2. Tell students that while they are working you will be circulating the *Attendance Form* so they can print their names beside their test booklet number.
	3. Read instructions for completing *Demographic Data Forms* printed on the top left corner. Direct students to print and shade in all the information requested on the *Demographic Data Form*. Remind students of the importance of accurately recording the program (English, Immersion or Français) they are now in.
	4. Collect the *Demographic Data Forms*.
	5. Read aloud the *Test Rules* to students (Appendix 1, p. 17).
	6. Read aloud *General Directions* to students (page *i* in test booklet).
	7. Remind students to:
	• pace themselves in writing the test.
	• use the Answer/Scoring Sheet for responding to multiple-choice questions.
	• check over their work to ensure that all questions have been attempted.
	8. **Instruct students to turn the page and begin. Record the start time on the *Supervising Teacher Report*.**
	During the test session
	1. Supervise the completion of the *Attendance Form* by ensuring that students print their name beside their test booklet number. (See sample *Attendance Form*, Appendix 3, p. 19). **Note:** This is the only record connecting students' identities to their *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* booklet.
	2. Verify that all the information on each *Demographic Data Form* is accurately printed and shaded. Place all used forms in numerical order followed by any unused forms in the labelled envelope provided. **Do not use elastic bands, staples or paper clips on these forms as they may damage the sheets.** Do not seal the envelope.
	3. Ensure that each student has recorded the correct booklet number by verifying that it corresponds to the student name printed on the *Attendance Form*.
	4. Half-way through Part 1 of the test session, advise students of the amount of time remaining by writing the remaining time on a board.

	5. After 120 minutes, ask the students to leave their calculators on the floor for the remainder of the test session. Students should keep their Part 1 booklet for the entire testing session. If they wish, they may do additional work/review on Part 1 questions, even though they will not have access to their calculators when Part 2 is distributed.
	6. Distribute Part 2 test booklets **(cross-reference the name and booklet number on the Attendance Forms with the numbers on the test booklets. Students must have their assigned number on both Part 1 and Part 2 booklets.)**
	7. **Instruct students to turn the page and begin. Record the start time on the *Supervising Teacher Report.***
	8. Half-way through Part 2 of the test session, advise students of the amount of time remaining by writing the remaining time on a board.
	Ending the test session
	1. Have students place their *Answer/Scoring Sheets* inside the front cover of the test booklet. Collect test booklets.
	2. As students submit their test materials, ensure that each test booklet number matches the information on the *Answer/Scoring Sheet*, the *Attendance Form* and the *Demographic Data Form* (e.g., cross-reference student name and test booklet number). Each student must have the same booklet number on all test materials.
	3. Place a check "✓" on the *Attendance Form* in the "Booklet Submitted to LMC for Marking" column.
	4. **Record the time students finish writing the test on the *Supervising Teacher Report*.** Do not include extra time given for adaptations.
	5. Stack the test booklets in numerical order.
	6. If marking is to be conducted at a central location, place all used *Answer/Scoring Sheets* in numerical order followed by any unused forms. Place these forms in the labelled envelope provided. **Do not use elastic bands, staples or paper clips on these forms as they may damage the sheets.** Do not seal the envelope.
	7. Complete the *Supervising Teacher Report*. Note any unusual circumstances on this report.
	8. Ensure that the number of check marks in the "Booklet Submitted to LMC for Marking" column on the *Attendance Forms* matches your count of the booklets and *Answer/Scoring Sheets* for submission.
	9. Complete and check the *Student Registration Update* form. Use the reason codes (Appendix 4, p. 20) provided.
	10. Make one or two copies of the *Attendance Form(s)* and *Supervising Teacher Report(s)*. One copy must be retained by the school and the other forwarded to the local marking site, if a centralized model is used.
	11. Collect the *Principal Feedback Form*.
	12. Complete the *Supervising Teacher Feedback Form*.
	13. Return all test booklets and other test-related materials to the **secure area** in the school until commencement of marking or until materials are to be forwarded to the local marking site.
	14. Send materials according to the instructions on the following pages.

Submitting Materials on January 22, 2002 (the end of the test)

It is essential that the following materials are packaged and forwarded to the local marking site immediately after completion of the test if a centralized model is used.

- **All used** test booklets;
- **All used** *Answer/Scoring Sheets*;
- Copy of the *Attendance Forms*; and
- Copy of the *Supervising Teacher Report*.

All other documentation should be verified and returned to the Assessment and Evaluation Branch after the test session. Use the envelope provided to return:

- *Attendance Forms*;
- **All used** *Demographic Data Forms* (in envelope provided);
- *Student Registration Update*;
- *Supervising Teacher Report*;
- *Supervising Teacher Feedback Form*;
- *Principal Feedback Form*; and
- **All** damaged or misprinted booklets.

Schools in the City of Winnipeg return their envelopes to the Assessment and Evaluation Branch in the following way. Internet Courier is scheduled to pick up envelopes between **1:00 p.m. and 4:00 p.m.** on **January 22, 2002**. If your envelope will not be ready for pick up during these hours, call Internet Courier (668-5552) to make alternate arrangements.

Schools outside the City of Winnipeg return their envelopes to the Assessment and Evaluation Branch by taking them to the post office. See page 13 for instructions on how to use the bill of lading.

Submitting Materials by February 6, 2002 (after local marking)

When local marking is done at a central location, the Local Marking Co-ordinator is responsible for sending the materials to Assessment and Evaluation. When local marking is done at the classroom level, the teacher is responsible for sending the materials to Assessment and Evaluation.

After local marking has been completed, prepare a package for return to Assessment and Evaluation no later than February 6, 2002. Use the set of return labels enclosed with the requested sample of test booklets.

The package must include:

- **All** *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* scan sheets;
- Test booklets requested by the department for the feedback marking;
- *Sample Request List*; and
- *Local Marking Co-ordinator Feedback Form*.

Jurisdictions within the City of Winnipeg send packages directly to the Assessment and Evaluation Branch via **Internet Courier (668-5552).** The person placing the call should quote **account number ED 502,** give the jurisdiction name, address, and the name of the contact person.

Jurisdictions outside the City of Winnipeg return their package by Xpresspost. It is very important to package all materials in **one box or mailing envelope** whenever possible.

After the package(s) is (are) sealed, affix the return labels supplied.

To utilize the **bill of lading**:

1. Sign the "Sender's Signature" section of the bill of lading at the "X".
2. Take your sealed envelope/package and bill of lading to the post office.
3. The postal clerk will fill in the weight of the envelope/package and prepare the envelope/package for mailing. Ask the postal clerk for the customer receipt and keep it as a record of shipment. If it is necessary to ask Canada Post to trace the envelope/package, your school will be contacted and asked to provide the item number.

We have provided sufficient materials for the return of **one** envelope/package. If you must send a second envelope/package, ask the postal clerk for an additional bar code label to attach to the second envelope/package. Ensure that the bar code number is recorded in the Item ID column. Note that the second envelope/package must be completely and accurately addressed to the Assessment and Evaluation Branch.

Appendices

Appendix 1: Test Rules

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

The supervising teacher is expected to read these rules aloud to students at the beginning of the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* – Session 1:

- Check the title of your test booklet carefully.
- During this test session, do not proceed until instructed to do so.
- If you receive a damaged or imperfectly printed booklet, return it to receive another booklet.
- You are expected to remain in the test room for the allotted time for Part 1 and until you complete Part 2. (You may leave before that time in exceptional circumstances such as illness, when accompanied by a supervisor.)
- At the beginning of the session you should have only a Part 1 test booklet, a Demographic Form, an HB pencil, eraser, ruler and your calculator (scientific or graphing – for Part 1 only). No other materials are permitted. Do not exchange test materials.
- Your calculator must remain on the floor throughout the administration of Part 2. You may keep your Part 1 booklet for the entire session and go back through Part 1 if you wish but you may not use your calculator.
- You will not give assistance to, nor receive assistance from, another student. If you require something during the test, raise your hand and I will come to you.
- Do not ask questions or request explanations with regard to the content of the questions on the test.
- Do **not** identify yourself or your school, teacher or town in your test booklet.
- When I ask you to stop writing, put your pencil down immediately.
- You may not discard any materials. The test booklet must remain intact unless otherwise instructed. You may **not** leave the room with any materials pertaining to the test.

Appendix 2: Sample Generic Demographic Data Form

Appendix 3: Sample Attendance Form

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ATTENDANCE FORM—JANUARY 2002

SENIOR 4 PRE-CALCULUS MATHEMATICS (40S) STANDARDS TEST

Booklet Number	Student Name (*Please Print*)	Booklet Submitted to LMC* for Marking (*check* ✓)	
		Part 1	Part 2
30001	Karen Masi	✓	*went home sick*
30002	Dan Carter	✓	✓
30003	Michael Smith	✓	✓
30004	DAMAGED		
30005	Mary Jones (also 30008)	✓	✓
30006	Shelley Bracken	✓	✓
30007	UNUSED		
~~30008~~	Used with 30005 (damaged)		
~~30009~~	Used in another classroom		
~~30010~~	Used in another classroom		

NOTE: Record any absences on the *Student Registration Update* form.

Myles Kaminsky

Supervising Teacher's Name (*please print*)

Please ensure that copies of these forms are retained for your records. You will need this form to link your locally marked test with a student's name.

*** Local Marking Co-ordinator**

Appendix 2: Sample Generic Demographic Data Form

Appendix 3: Sample Attendance Form

Page _____ of _____

ATTENDANCE FORM—JANUARY 2002

SENIOR 4 PRE-CALCULUS MATHEMATICS (40S) STANDARDS TEST

Booklet Number	Student Name (*Please Print*)	Booklet Submitted to LMC* for Marking (*check* ✓)	
		Part 1	Part 2
30001	Karen Masi	✓	*went home sick*
30002	Dan Carter	✓	✓
30003	Michael Smith	✓	✓
30004	DAMAGED		
30005	Mary Jones (also 30008)	✓	✓
30006	Shelley Bracken	✓	✓
30007	UNUSED		
30008	Used with 30005 (damaged)		
30009	Used in another classroom		
30010	Used in another classroom		

NOTE: Record any absences on the *Student Registration Update* form.

Myles Kaminsky

Supervising Teacher's Name (*please print*)

Please ensure that copies of these forms are retained for your records. You will need this form to link your locally marked test with a student's name.

* **Local Marking Co-ordinator**

Appendix 4: List of Reason Codes to Record Changes in Standards Test Student Registration Numbers

(Please use these codes to complete the Student Registration Update when returning to the Assessment and Evaluation Branch in January 2002.)

Absent	
01	arrived too late to write the test
02	did not write due to weather
03	family/personal reasons
04	medical/illness/accident
05	out-of-town activity (athletic/academic)
06	religious objections
07	student refused to write
08	unexcused
09	working
19	other (please specify)

Withdrawn	
22	dropped course
23	incarcerated/court
24	medical/illness/accident
25	family/personal reasons
26	poor attendance/progress
27	quit school
29	other (please specify)

Incomplete	
40	continuing course/program next semester
41	no work completed
49	other (please specify)

Errors in Registration	
20	changed program
21	deceased
50	already has credit
51	auditing the course
52	exchange student
53	registered twice
54	quarter system projected registration
55	registered in wrong grade
59	other (please specify)

Exemptions	
60	emotional/psychological condition or circumstances
61	"M" credit designation
62	"E" credit designation
63	"phase d'accueil—bloc intensif" designation
69	other (please specify)

Drop Ins	
70	forgotten on original test registration
71	late registration to course
72	transferred in from another school/province/country
79	other (please specify)

Migrancy	
90	transferred to another school within the province
91	transferred to another school outside of the province
99	other (please specify)

Pre-Calculus Mathematics (40S)

Standards Test

Part 1

January 2002

Manitoba
Education,
Training
and Youth

Pre-Calculus Mathematics (40S)
Standards Test

DESCRIPTION

Total Possible Marks: 100 **Time: 3 hours**

Part	Description	Time	Value
Part 1	• **Multiple choice:** 30 questions worth 1 mark each • **Long answer:** 13 questions worth a total of 40 marks Calculators (scientific or graphing) are permitted.	120 minutes	70%
Part 2	• **Short answer:** 10 questions worth 1 mark each • **Long answer:** 7 questions worth a total of 20 marks Calculators are not permitted.	60 minutes	30%

GENERAL DIRECTIONS

- Fill in the demographic form accurately and completely.
- Formulas are given on the tear-out "Formula Sheet" at the beginning of this test booklet. Please do not tear out any pages other than the "Formula Sheet".
- Read all instructions carefully.
- The blank pages at the back of each booklet may be used for rough work. No marks will be given for work done on these pages.
- Note that diagrams and graphs may not be drawn to scale.
- When Part 2 is distributed, you may keep Part 1 in case you wish to continue working on Part 1 questions (without a calculator).

Pre-Calculus Mathematics 40S Formula Sheet

$$\sin^2\theta+\cos^2\theta=1$$

$$\tan^2\theta+1=\sec^2\theta$$

$$1+\cot^2\theta=\csc^2\theta$$

$$P(n,r) \text{ or } {}_nP_r=\frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r=\frac{n!}{r!(n-r)!}$$

$$(x-h)^2+(y-k)^2=r^2$$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,\ a>b$$

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1,\ a>b$$

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

$$(x-h)^2=a(y-k)$$

$$(y-k)^2=a(x-h)$$

$$S=r\theta$$

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$A=P\left(1+\frac{r}{n}\right)^{nt}$$

$$A=Pe^{rt}$$

$$e\approx 2.71828$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\log_a(MN)=\log_a M+\log_a N$$

$$\log_a\left(\frac{M}{N}\right)=\log_a M-\log_a N$$

$$\log_a(M^n)=n\log_a M$$

$$\log_a M=\frac{\log_b M}{\log_b a}$$

$$t_n=t_1r^{n-1}$$

$$S_n=\frac{t_1(1-r^n)}{1-r}=\frac{t_1(r^n-1)}{r-1}$$

$$S_n=\frac{t_1-t_nr}{1-r}$$

$$S_\infty=\frac{t_1}{1-r},\ |r|<1$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\sin 2\alpha=2\sin\alpha\cos\alpha$$

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=1-2\sin^2\alpha=2\cos^2\alpha-1$$

$$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$$

Multiple-Choice Questions

Instructions

- There are 30 multiple-choice questions.
- Each question is worth 1 mark.
- Record your answers on the sheet provided.
- Read each question carefully.
- Examine all the choices before you decide on your answer.
- You may use the spaces beside each question for rough work.
- Answer all questions.
- Provide only one answer per question.
- There is no penalty for guessing.
- Calculators (scientific or graphing) are allowed for this part of the test (for the first 2 hours).

Multiple-Choice Questions

1. An angle coterminal with 20° is:

   A) –20°

   B) 160°

   C) 200°

   D) 380°

2. An angle of 10 radians is equivalent to an angle of:

   A) $\frac{\pi^\circ}{18}$

   B) 1800°

   C) $\frac{10^\circ}{\pi}$

   D) $\frac{1800^\circ}{\pi}$

3. On the interval $[0, 2\pi]$ the graphs of $y = \sin x + 1$ and $y = 2$ intersect in:

   A) 0 points

   B) 1 point

   C) 2 points

   D) more than 2 points

4. A solution for $\tan\theta = \frac{-1}{2}$ with $0 \le \theta \le 2\pi$ is approximately:

A) 0.464

B) 2.678

C) 3.605

D) 153.435

5. The expression $\frac{2\sin x}{\sin(2x)}$ is equivalent to:

A) 1

B) $2\csc x$

C) $\frac{2\sin x}{1-\cos^2 x}$

D) $\sec x$

6. On the interval $[0, 2\pi]$, the function $y = \sqrt{\frac{1}{\sin x}}$ is defined for:

A) $[0, 2\pi]$

B) $(0, 2\pi)$

C) $(0, \pi)$

D) $[0, \pi]$

7. If $P(\theta)$ is a point on the unit circle, then $P(24)$ lies in quadrant:

   A) I

   B) II

   C) III

   D) IV

8. A circle with a radius of 10 cm has a central angle of 108° intercepting an arc AB. The length of arc AB is:

   A) 6 cm

   B) 6π cm

   C) 1080 cm

   D) 1080π cm

9. If $\tan\theta = \frac{a}{b}$ and $\sec\theta = \frac{c}{b}$, then $\sin\theta$ is equivalent to:

   A) $\frac{a}{c}$

   B) $\frac{ac}{b^2}$

   C) $\frac{c}{a}$

   D) $\frac{b^2}{ac}$

10. What is the period of the function $y = \cos(2x)$?

A) $\frac{\pi}{2}$

B) π

C) 2π

D) 4π

11. A value of x for which the function $y = \csc x$ is undefined is:

A) $\frac{\pi}{2}$

B) π

C) $\frac{3\pi}{2}$

D) $\frac{\pi}{4}$

12. The point $(4, -5)$ is on the graph of $y = f(x)$.

On the graph $y = f(2x) - 3$, this point will be transformed to:

A) $(8, -8)$

B) $(8, -2)$

C) $(2, -2)$

D) $(2, -8)$

13. Which of the following functions has a period of 2 and a range of $[-4, 6]$?

A) $y = 5\cos(2\pi x) + 1$

B) $y = 5\cos(\pi x) + 1$

C) $y = 5\cos(\pi x) - 1$

D) $y = 5\cos\left(\frac{\pi}{2}x\right) - 1$

14. Find the value of x:

$$\log_2(\log_3 x) = 2$$

A) 2^3

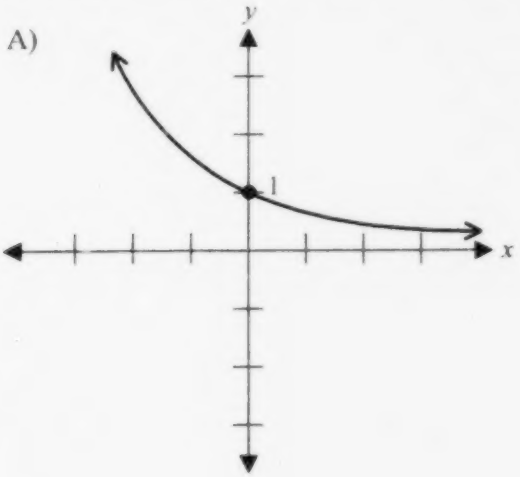
B) 3^2

C) 3^4

D) 4^3

15. Given $f(x) = a^x$, where $a > 1$, which of the following graphs best represents $y = f^{-1}(x)$?

A)



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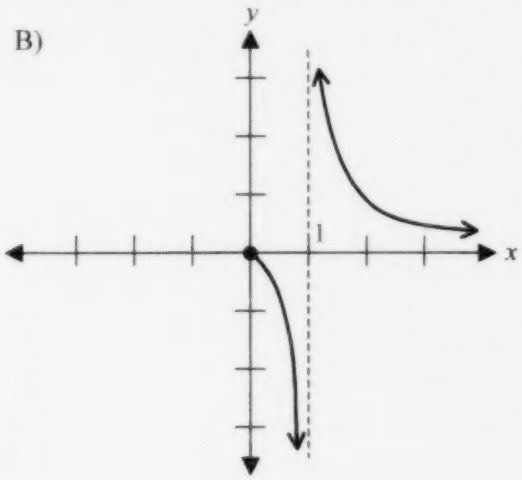
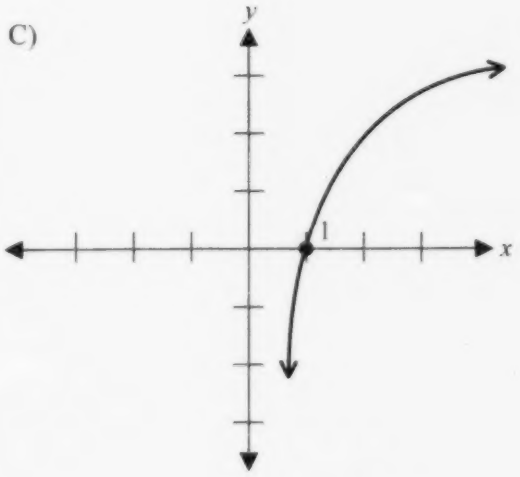
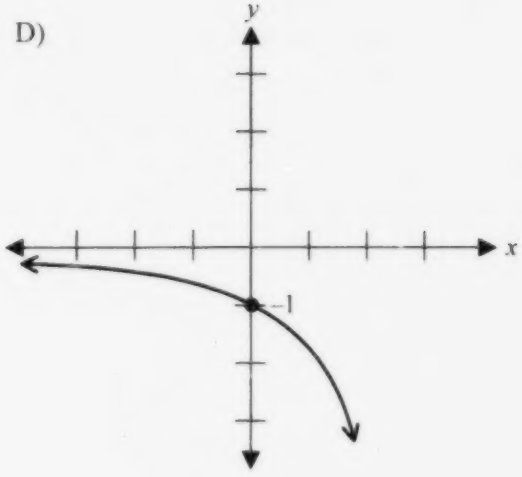


C)



D)



16. For positive values of x the expression $\log_3 \dfrac{x^2(x+7)}{\sqrt{x}}$ is equivalent to:

A) $2\log_3 x + \log_3(x+7) - \log_3\left(\dfrac{x}{2}\right)$

B) $\dfrac{1}{2}\log_3 x + \log_3(x+7) - 2\log_3 x$

C) $2\log_3 x + \log_3(x+7) - \dfrac{1}{2}\log_3 x$

D) $\log_3(x^3) + \log_3(7x^2) - \dfrac{1}{2}\log_3 x$

17. The expression $\ln e^5 + 2\ln e^{-3}$ is equivalent to:

A) -30

B) -1

C) $\frac{5}{8}$

D) $\frac{41}{8}$

18. John's uncle invests $5000 when John is born, with the condition that it not be withdrawn until John is 30 years old.

Assuming the interest rate is 8% compounded semi-annually, which of the following expressions would represent the value of the investment after 30 years?

A) $5000(1+0.08)^{30}$

B) $5000(1+0.04)^{60}$

C) $5000(1+0.04)^{30}$

D) $5000(1+0.08)^{60}$

19. In how many ways can 6 different coloured beads be arranged on a bracelet?

A) $\frac{5!}{2}$

B) $\frac{6!}{2}$

C) $5!$

D) $6!$

20. An expression that is equal to n is:

A) $\dfrac{(n+1)!}{n!}$

B) $\dfrac{2n!}{2!}$

C) $\dfrac{n!}{(n+1)!}$

D) $\dfrac{n!}{(n-1)!}$

21. The term containing x^{52} in the expansion of $(2x+y)^{100}$ is the:

A) 48^{th} term

B) 49^{th} term

C) 50^{th} term

D) 51^{st} term

22. In how many ways can all the letters in the word C A L C U L U S be arranged if each arrangement begins and ends with the letter C?

A) $8!$

B) $6!$

C) $\dfrac{6!}{2!2!}$

D) $\dfrac{8!}{2!2!2!}$

23. The domain of the relation $(x+1)^2+(y-2)^2=4$ is:

A) $[-1,3]$

B) $[-5,3]$

C) $[0,4]$

D) $[-3,1]$

24. The range of the relation $\frac{y^2}{4}-\frac{x^2}{9}=1$ is:

A) $(-\infty,-2]\cup[2,\infty)$

B) $(-\infty,-3]\cup[3,\infty)$

C) $[-2,2]$

D) $[-3,3]$

25. For the ellipse $\frac{(x-2)^2}{16}+\frac{(y+3)^2}{25}=1$ what is the length of the major axis?

A) 4

B) 5

C) 8

D) 10

26. In a school there are 100 Senior students and 285 Junior students.

Of these students, 54 Seniors are girls and 165 Juniors are girls.

If a student is chosen at random, what is the probability that the student is a Junior girl?

A) $\frac{165}{385}$

B) $\frac{165}{285}$

C) $\frac{165}{220}$

D) $\frac{165}{219}$

27. A jar contains ten red cards numbered from 1 to 10, and ten green cards numbered from 11 to 20.

A card is chosen from the jar at random.

What is the probability of choosing either a red card or a card with a prime number?

A) $\frac{1}{2}$

B) $\frac{7}{10}$

C) $\frac{4}{5}$

D) $\frac{9}{10}$

28. A box contains 10 red marbles and 6 blue marbles.
Two marbles are drawn in succession without replacement.
What is the probability that the marbles are different colors?

A) $\frac{1}{2}$

B) $\frac{1}{4}$

C) $\frac{15}{32}$

D) $\frac{15}{64}$

29. The probability that an event will occur on a single trial is $\frac{2}{7}$.
What is the probability that the event will not occur on either of two successive trials?

A) $\frac{2}{5}$

B) $\frac{4}{49}$

C) $\frac{25}{49}$

D) $\frac{45}{49}$

30. Find the sum of

$$100 - 20 + 4 - \frac{4}{5} + \ldots$$

A) $\frac{500}{9}$

B) 125

C) 500

D) $\frac{250}{3}$

Long-Answer Questions

Instructions

- There are 13 questions for a total of 40 marks.
- Calculators (scientific or graphing) are allowed for this part of the test (for the first 2 hours).
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions should include an explanation of how your final answer is obtained.
- Full marks will NOT be awarded for your final answer only.
- Consideration will be given to neatness, organization, and clarity of expression.

DO NOT USE

Long-Answer Questions

31. If $\sin\theta = \frac{2}{5}$ and θ is in quadrant II, find the exact value of $\tan\theta$. 101

[3 marks]

32. State the interval(s) where $\sin\theta > \cos\theta$, for $0 \le \theta \le 2\pi$. 102

Justify your answer with an explanation or a clearly labelled diagram.

[3 marks]

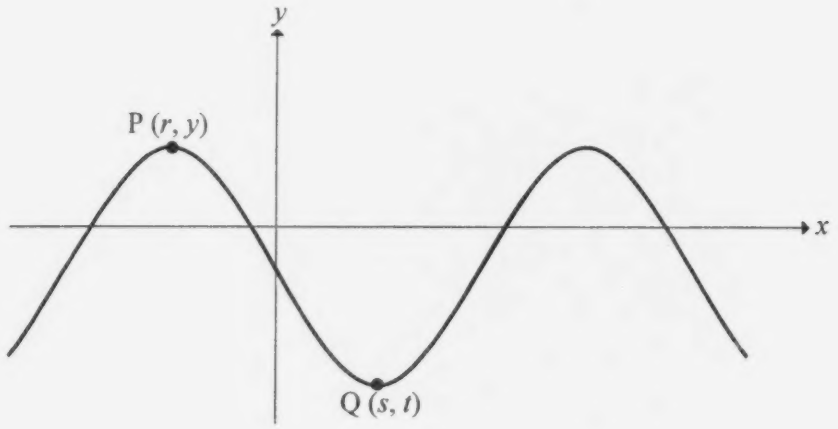
DO NOT USE

33. The graph of $y = 3\cos[2(x+\frac{\pi}{4})]-1$ is sketched below. 103

The point P is a maximum and the point Q is a minimum.

Determine the exact values of the coordinates r, s and t.

[3 marks]



r _____

s _____

t _____

34. A mathematics class consists of 7 girls and 6 boys. 104

a) In how many ways can a group of 8 students be selected from this class?
State your final answer as a whole number.

[1 mark]

b) How many groups of 8 students consist of 5 girls and 3 boys? 105
State your final answer as a whole number.

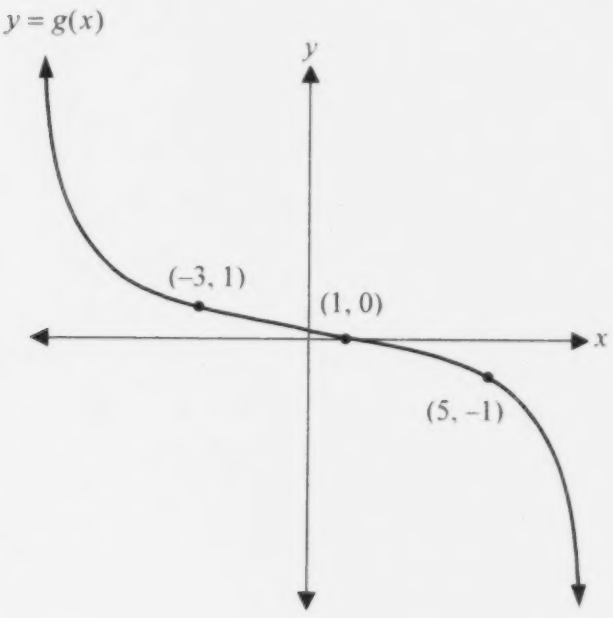
[2 marks]

DO NOT USE

35. Given the graph of $y = g(x)$:

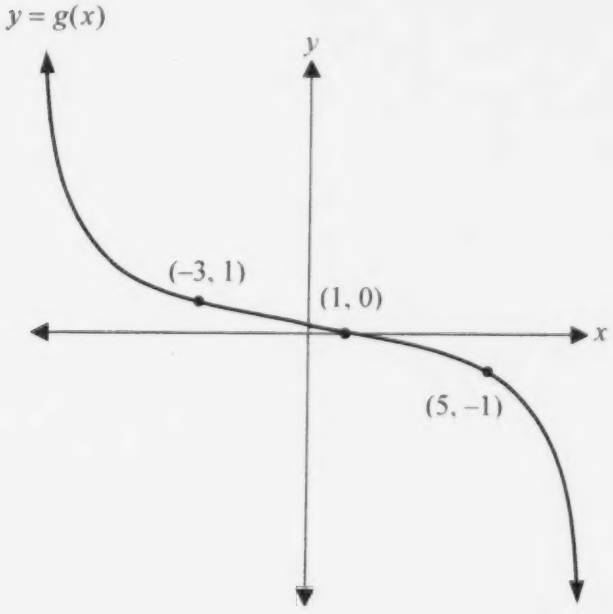
a) On the axes provided, sketch the graph of $y = -g(x) + 3$. 106

[2 marks]



b) On the axes provided, sketch the graph of $y = g^{-1}(x)$. 107

[1 mark]



DO NOT USE

108

36. Find the exact value for $\sec\left(\frac{\pi}{12}\right)$.

[4 marks]

37. Prove: $\tan x = \dfrac{1 - \cos 2x}{\sin 2x}$

[4 marks]

DO NOT USE

109

DO NOT USE

38. In a first-year university calculus class, 70% of the students have taken calculus in high school.

110

The pass rate in university calculus for students who had taken high school calculus is 80%.

The pass rate in university calculus for students who had not taken high school calculus is 40%.

a) If a student took high school calculus, what is the probability of not passing university calculus?

[1 mark]

111

b) If a student passes university calculus, what is the probability that he took calculus in high school?

[3 marks]

DO NOT USE

39. If $\log_a 2 = x$ and $\log_a 5 = y$, find an algebraic expression for $\log_a\left(\frac{\sqrt[3]{10}}{2}\right)$ in terms of x and y.

112

[3 marks]

40. Solve for x algebraically.
Show all steps.

113

$$\ln\left(e^{\sqrt{x}}\right) = 8$$

[2 marks]

DO NOT USE

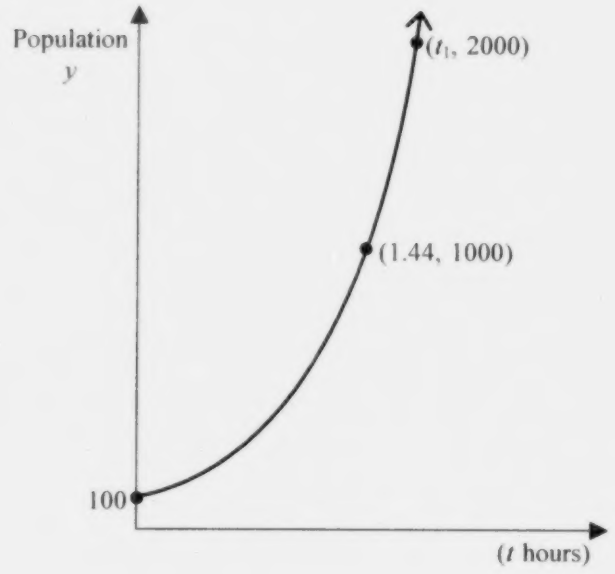
114

41. The population growth of a bacterial culture follows an exponential curve $y = 100e^{kt}$ as shown in the diagram.

Find the value of t_1, the time that the population reaches 2000.

Round your final answer to two decimal places.

[3 marks]



DO NOT USE

42. Find a simplified expression for the fourth term in the expansion of $\left(x^2 - \frac{2}{x}\right)^{10}$. 115

[3 marks]

43. In how many ways can 7 people be seated at a round table if 2 people refuse to sit next to each other? 116

[2 marks]

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

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Pre-Calculus Mathematics (40S)

Standards Test

Part 2

January 2002

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Short-Answer Questions

Instructions

- There are 10 short-answer questions.
- Each question is worth 1 mark.
- Calculators are NOT allowed for this part of the test.
- Write your final answer in the space provided.
- Work need not be shown in order to obtain full marks for a correct answer. However, in the case of an incorrect final answer, part marks may be awarded only if work is shown.

DO NOT USE

Short-Answer Questions

44. What is the value of $\cos^2(30°) + \sin^2(60°)$?

117

[1 mark]

45. Express 13° in radian measure.
State your answer as an exact value.

118

[1 mark]

DO NOT USE

46. What is the range of the function $y = |2\cos x|$?

119

[1 mark]

47. If the x-intercept of $y = f(x)$ is 4, what is the value of the x-intercept for $y = 2f(-x)$?

120

[1 mark]

48. What is the maximum value of $3(\cos^2 x - \sin^2 x)$?

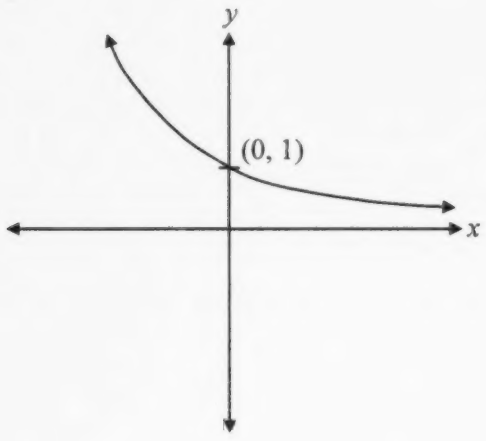
121

[1 mark]

DO NOT USE

49. The graph of the exponential function $y = a^x$ is shown below. State a possible value for a. 122

[1 mark]



50. Solve for x: 123

$$125 = 5^{2x-1}$$

[1 mark]

DO NOT USE

51. There are 6 finalists in the 100-meter dash at the Snow Lake Track Meet. 124
In how many different ways can first and second place medals be awarded?
Assume that there are no ties.
State your answer as a whole number.

[1 mark]

52. What type of conic is represented by the following equation? 125

$$x^2 + 4x - 2y + 12 = 0$$

[1 mark]

53. A bag contains x red balls and y yellow balls. 126
A ball is drawn at random from the bag.
Find an algebraic expression to represent the probability that the ball is red.

[1 mark]

Long-Answer Questions

Instructions

- There are 7 questions worth a total of 20 marks.
- Calculators are NOT allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Full marks will NOT be awarded for your final answer only.
- Consideration will be given to neatness, organization, and clarity of expression.

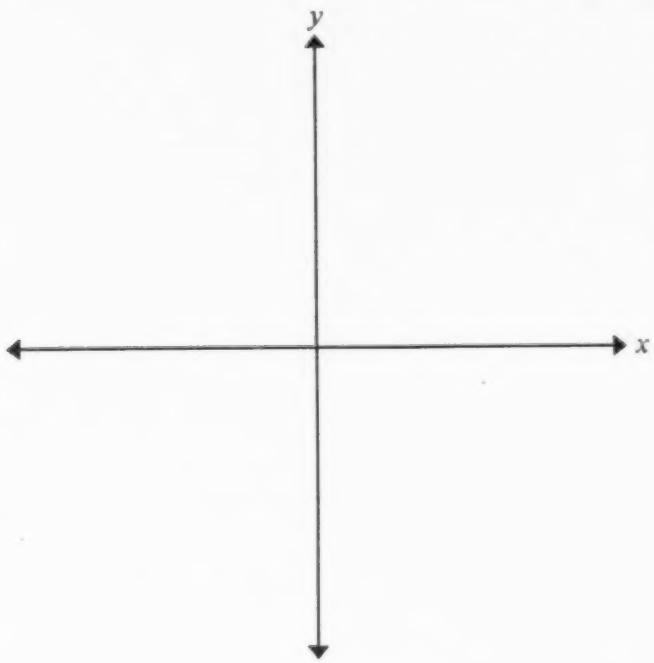
DO NOT USE

Long-Answer Questions

54. Sketch a clearly labelled graph of $y = \sec x$ showing at least one period. 127

State the range of this function.

[3 marks]



55. Compare the graphs of the relations $x^2 = y$ and $(x-2)^2 = y+4$. 128
(State one similarity and one difference.)

[2 marks]

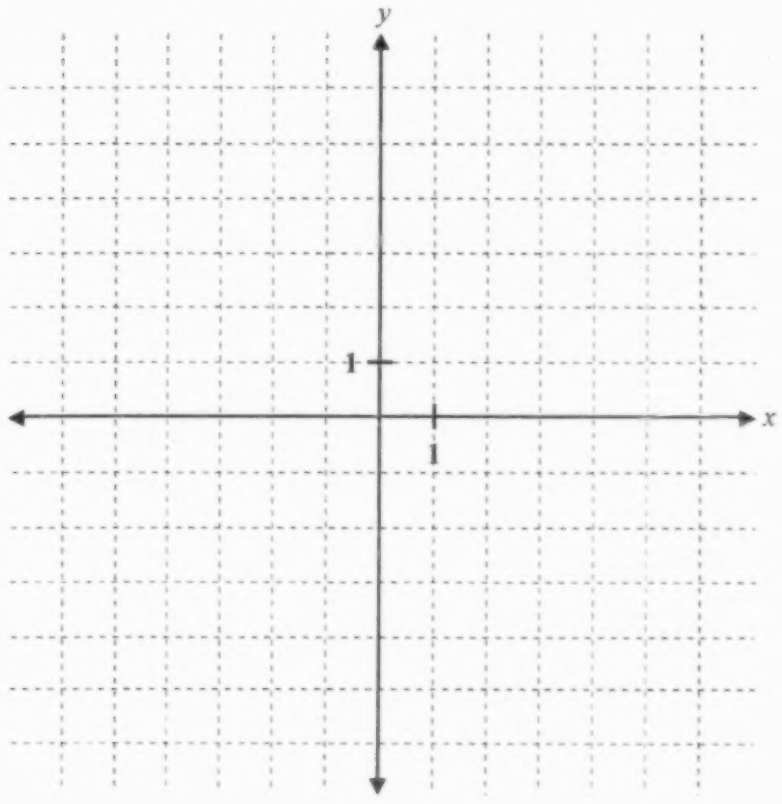
DO NOT USE

56. a) Write the equation $9x^2 - 18x + 4y^2 + 16y - 11 = 0$ in standard form. 129

[3 marks]

b) Sketch a clearly labelled graph of the above equation. 130

[1 mark]



DO NOT USE

57. Jack and Jill are students in a class of 20.

A committee of 5 students is to be selected from the class.

Find the probability that both Jack and Jill will be selected for the committee.

Express your answer as a number in its simplest form.

131

[3 marks]

58. Find the exact value(s) of x:

$\sin^2 x - 3\cos x = 3$, where $0 \le x < 2\pi$.

132

[4 marks]

DO NOT USE

59. Find t_8 in a geometric sequence where $t_1 = 16$ and $t_4 = 2$. 133

[2 marks]

60. Evaluate $\sum_{i=2}^{4} (i+1)^2$. 134

[2 marks]

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Pre-Calculus Mathematics (40S)
Standards Test

Marking Package

January 2002

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Education,
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Part 2: Long-Answer Questions 55

General Marking Instructions

Please make no marks in the *Student Response Booklets*.

Please ensure that:

- the student test booklet number and the number on this sheet are identical;
- **only a pencil is used to complete the *Answer/Scoring Sheets*, by both students and markers**;
- the totals of each of the four parts are written at the bottom;
- each student's final result is recorded, by test booklet number, on the corresponding ***Answer/Scoring Sheet*** (computer scan sheet);
- the grand total is written in the appropriate location;
- the *Answer/Scoring Sheet* is complete; and
- a photocopy has been made for school records.

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education, Training and Youth (for more information see the *Administration Manual for Supervising Teachers*.

Scoring the Multiple-Choice Questions

Please follow the instructions on the ***Answer Key for Multiple-Choice Questions*** on page 5.

Scoring the Short-Answer and Long-Answer Questions

The remainder of Senior 4 Pre-calculus Mathematics Standards Test is composed of short-answer questions and long-answer questions. Short-answer questions are worth 1 mark each and long-answer questions are worth 2 to 4 marks each.

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student's response must be complete and correct. Otherwise, 1 mark must be deducted for each concept error, and ½ mark must be deducted for a calculation or arithmetic error. Where alternative answering methods are possible, the *Marking Key* attempts to address the most common solutions.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education, Training and Youth at the earliest opportunity to advise and/or seek assistance. If required, we will connect you to an advisor. Also, please be sure to notify us before making any modifications to the answer keys or scoring rubrics. Contact person: Wenda Dickens at 945-6774 (1 800 282-8069, extension 6774).

SCORING GUIDELINES

Answer Key for Multiple-Choice Questions

Answer Key for Multiple-Choice Questions

If you are using this sheet by punching out holes corresponding to the correct answers and overlaying this sheet on the *Answer/Scoring Sheet*, then

1) check the student's responses for multiple bubbles first (questions with multiple bubbles are to be scored as "0");

2) overlay this page on the *Answer/Scoring Sheet*; and

3) count the number of correct responses, excluding questions with multiple responses, if any.

You may also score the multiple-choice questions by making a transparency of this page and overlaying it on the *Answer/Scoring Sheet*.

Remember to write the total score for the multiple-choice questions at the bottom of the *Answer/Scoring Sheet*.

MULTIPLE-CHOICE ANSWER / RÉPONSES AUX QUESTIONS À CHOIX MULTIPLE

Fill in the best answer for each question.
Choisir la meilleure réponse à chaque question.

PART 1 / PARTIE 1

1	D	11	B	21	B
2	D	12	D	22	C
3	B	13	B	23	D
4	B	14	C	24	A
5	D	15	C	25	D
6	C	16	C	26	A
7	D	17	B	27	B
8	B	18	B	28	A
9	A	19	A	29	C
10	B	20	D	30	D

SCORING GUIDELINES

Marking Key
including student exemplars

Part 1—Long Answer

Question No. 31

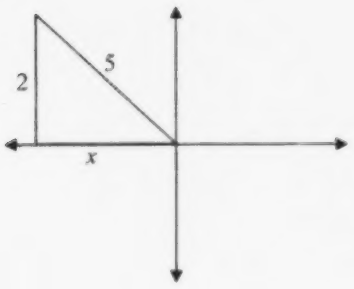
Outcome: A2

Question

31. If $\sin\theta = \frac{2}{5}$ and θ is in quadrant II, find the exact value of $\tan\theta$. **3 marks**

Solution

Method 1:



$x^2 + 2^2 = 5^2$

$x^2 = 21$

$x = \pm\sqrt{21}$

} 1 mark for method

since θ is in quadrant II,

$x = -\sqrt{21}$

$\therefore \tan\theta = -\frac{2}{\sqrt{21}}$ OR $-\frac{2\sqrt{21}}{21}$

} 1 mark for negative sign
1 mark for consistent value of tan θ

Method 2:

$\sin^2\theta + \cos^2\theta = 1$

$\left(\frac{2}{5}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{21}{25}$

$\cos\theta = \pm\frac{\sqrt{21}}{5}$

} 1 mark for method

since θ is in quadrant II,

$\cos\theta = -\frac{\sqrt{21}}{5}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{2}{5}}{-\frac{\sqrt{21}}{5}} = -\frac{2}{\sqrt{21}}$ OR $-\frac{2\sqrt{21}}{21}$

} 1 mark for negative sign
1 mark for consistent value of tan θ

Question No. 31

Outcome: A2

Question

31. If $\sin\theta = \frac{2}{5}$ and θ is in quadrant II, find the exact value of $\tan\theta$. **3 marks**

Solution

Method 3:

$\sin^2\theta + \cos^2\theta = 1$

$(0.4)^2 + \cos^2\theta = 1$

$\cos^2\theta = 0.84$

$\cos\theta = \pm\sqrt{0.84}$

} 1 mark for method

since θ is in quadrant II, $\cos\theta = -\sqrt{0.84}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.4}{-\sqrt{0.84}} = -\frac{0.4}{\sqrt{0.84}}$

} 1 mark for negative sign
1 mark for consistent value of tan θ

Note(s):

- if $\frac{-2\sqrt{21}}{21}$ *or* $\frac{-2}{\sqrt{21}}$ is given as well as –0.4364, **deduct** ½ mark
- if –0.4364 is given (without a radical value shown) **deduct** 1 mark
- if only calculator work is shown, with a decimal answer, allocate a maximum of 1 mark

EXEMPLAR

31. If $\sin\theta = \frac{2}{5}$ and θ is in quadrant II, find the exact value of $\tan\theta$.

$$\tan\theta = -\frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}}$$

$$\tan\theta = -\frac{2\sqrt{21}}{21}$$

Mark:	**2 out of 3**
Note(s):	• 1 mark for negative sign • 1 mark for correct value of $\tan\theta$ • deducted 1 mark for no method

Question No. 32

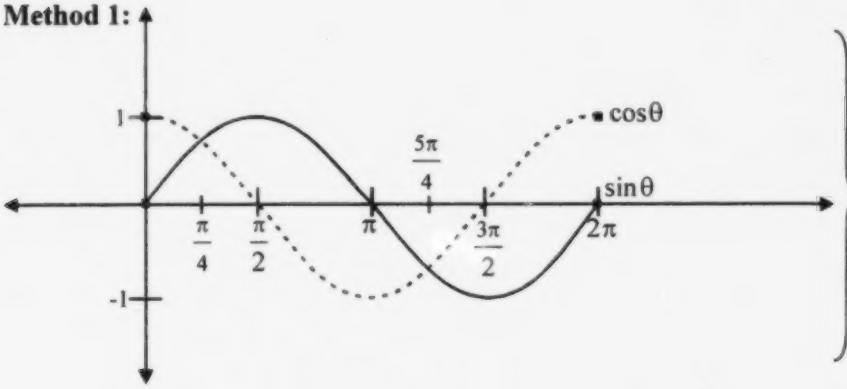
Outcomes: A3, A4, A6

Question

32. State the interval(s) where $\sin\theta > \cos\theta$, for $0 \le \theta \le 2\pi$.
Justify your answer with an explanation or a clearly labelled diagram. **3 marks**

Solution

Method 1:



½ mark for **each** correctly drawn graph

1 mark for reasonably accurate points of intersection of graphs

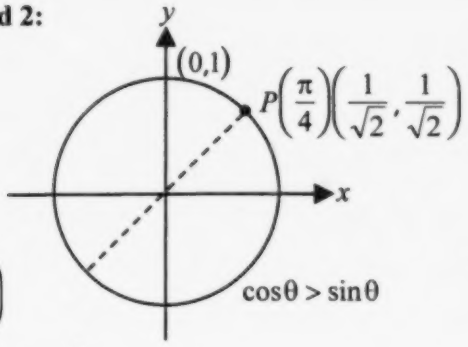
(intersection points need not be labelled)

interval is $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ OR $\frac{\pi}{4} < \theta < \frac{5\pi}{4}$

OR (0.7854, 3.927)

} 1 mark for consistent answer

Method 2:



$P\left(\frac{5\pi}{4}\right)$

$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Students should give some form of an explanation for **each** quadrant, for example:

- between $\frac{\pi}{4}$ and $\frac{\pi}{2}$ student must indicate that positive y segment length is greater than positive x segment length
- In quad 2, student should state that a positive number is greater than a negative number
- In quad 3 between π and $\frac{5\pi}{4}$ students should state $\sin\theta$ is less negative than $\cos\theta$

2 marks for explanation

interval is $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ OR $\frac{\pi}{4} < \theta < \frac{5\pi}{4}$

OR (0.7854, 3.927)

} 1 mark for consistent answer

Question No. 32

Outcomes: A3, A4, A6

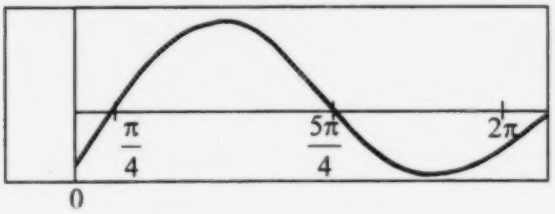
Question

32. State the interval(s) where $\sin\theta > \cos\theta$, for $0 \le \theta \le 2\pi$.
Justify your answer with an explanation or a clearly labelled diagram. **3 marks**

Solution

Method 3:

On graphing calculator, graph $y = \sin\theta - \cos\theta$ on interval $[0, 2\pi]$



2 marks for explanation

Student should explain that the portion above the x-axis is the solution.

interval is $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ OR $\frac{\pi}{4} < \theta < \frac{5\pi}{4}$

OR (0.7854, 3.927)

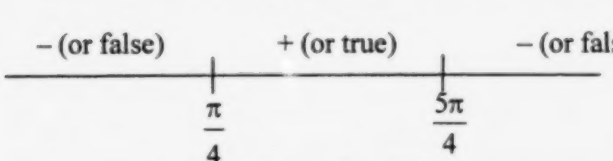
1 mark for consistent answer

Method 4:

Solve: $\sin\theta = \cos\theta$

$\theta = \frac{\pi}{4}, \theta = \frac{5\pi}{4}$

- correctly labelled sign diagram for the inequality $\sin\theta > \cos\theta$



2 marks for explanation

- solution $\frac{\pi}{4} < \theta < \frac{5\pi}{4}$ OR $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ OR (0.7854, 3.927)

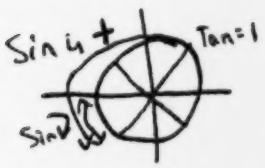
1 mark for correct interval

Note(s):

- **deduct** ½ mark if interval is fully or partially closed such as $\frac{\pi}{4} \le \theta \le \frac{5\pi}{4}$, $\frac{\pi}{4} < \theta \le \frac{5\pi}{4}$, OR $[0.7854, 3.927)$ OR $(0.7854, 3.927]$
- **deduct** ½ mark for correct answer given in degrees

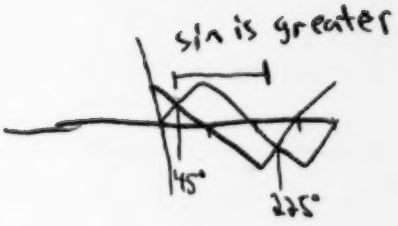
EXEMPLAR

32. State the interval(s) where $\sin\theta > \cos\theta$, for $0 \le \theta \le 2\pi$.
Justify your answer with an explanation or a clearly labelled diagram.



$\text{Tan} = \frac{\text{Sin}}{\text{Cos}}$ if tan is greater than 1
Sin is greater

45° to 225°



Mark:	**1½ out of 3**
Note(s):	• deducted ½ mark for answer in degrees • deducted ½ mark for incorrect statement about $\tan\theta$ • deducted ½ mark for implied inclusion of endpoints

Question No. 33

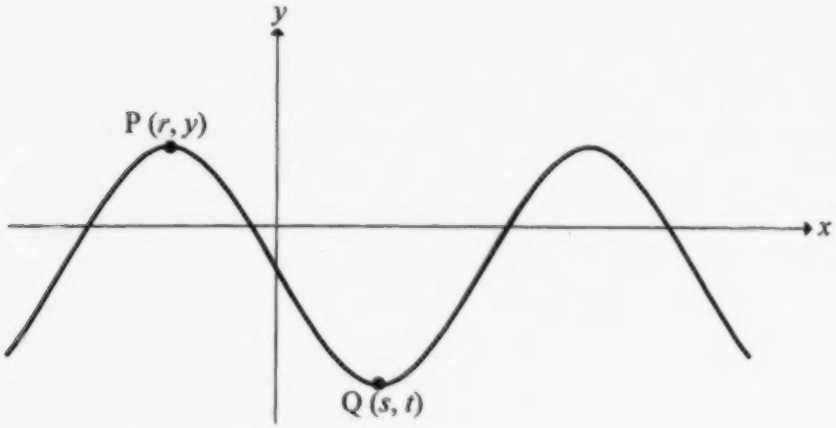
Outcome: B7

Question

33. The graph of $y = 3\cos[2(x + \frac{\pi}{4})] - 1$ is sketched below.

The point P is a maximum and the point Q is a minimum.
Determine the exact values of the coordinates r, s and t.

3 marks



Solution

$r = \frac{-\pi}{4}$ } 1 mark

$s = \frac{\pi}{4}$ } 1 mark

$t = -4$ } 1 mark

Note(s):

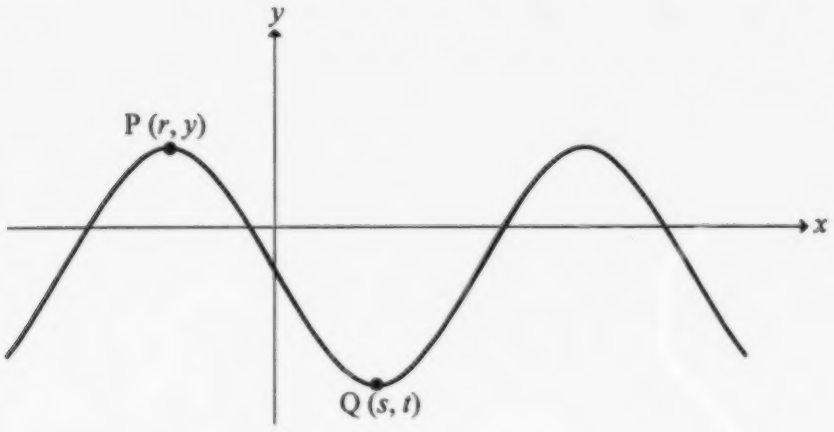
- only exact values are acceptable
- do not award ½ marks for this question

EXEMPLAR

33. The graph of $y = 3\cos[2(x + \frac{\pi}{4})] - 1$ is sketched below.

The point P is a maximum and the point Q is a minimum.

Determine the exact values of the coordinates r, s and t.



r $-\pi/4$

s $\pi/3$

t -2

Mark:	**1 out of 3**
Note(s):	• only one correct value, r.

Question No. 34

Outcome: E3

Question

34. A mathematics class consists of 7 girls and 6 boys.

a) In how many ways can a group of 8 students be selected from this class?
State your final answer as a whole number.

b) How many groups of 8 students consist of 5 girls and 3 boys?
State your final answer as a whole number.

3 marks

Solution

a) ${}_{13}C_8 = \dfrac{13!}{5!8!}$ } ½ mark

$= 1287$ } ½ mark for correct answer

b) There are ${}_7C_5$ ways of choosing the girls
There are ${}_6C_3$ ways of choosing the boys } 1 mark

$\therefore$ Total number of groups $= ({}_7C_5) \times ({}_6C_3)$ ½ mark

$= 21 \times 20$

$= 420$ ½ mark for correct answer

Note(s):

a) only give ½ mark if answer given as ${}_{13}C_8$ or $\dfrac{13!}{5!8!}$ or 1287

b)

- one mark maximum if ${}_7C_5 + {}_6C_3$
- if answer given as ${}_7C_5 \times {}_6C_3 = 420$ give full marks
- if answer given as ${}_7C_3 \times {}_6C_5 = 210$ give 1 mark

EXEMPLARS

34. A mathematics class consists of 7 girls and 6 boys.

a) In how many ways can a group of 8 students be selected from this class?
State your final answer as a whole number.

13P8 there are 518918400 ways
the students can be chosen.

Mark: **0 out of 1**
Note(s): • changed combination to a permutation

b) How many groups of 8 students consist of 5 girls and 3 boys?
State your final answer as a whole number.

$_7P_5 = 2520$
$_6P_3 = 120$ 302400 groups consist of 5 girls and 3 boys.

Mark: **1 out of 2**
Note(s): • deducted 1 mark for using permutations instead of combinations
• correctly multiplied terms

Question No. 35

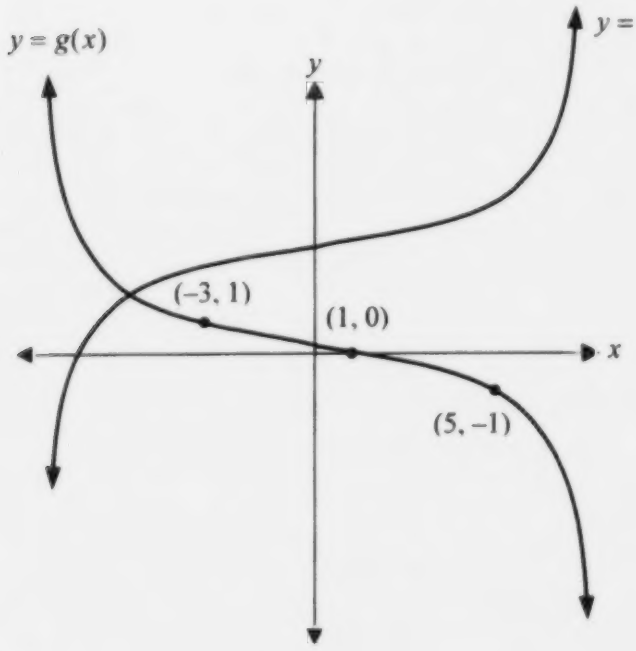
Outcomes: B3, B6

Question

Given the graph of $y = g(x)$:

a) On the axes provided, sketch the graph of $y = -g(x) + 3$. **2 marks**

Solution



1 mark for vertical reflection

1 mark for vertical translation

Question No. 35

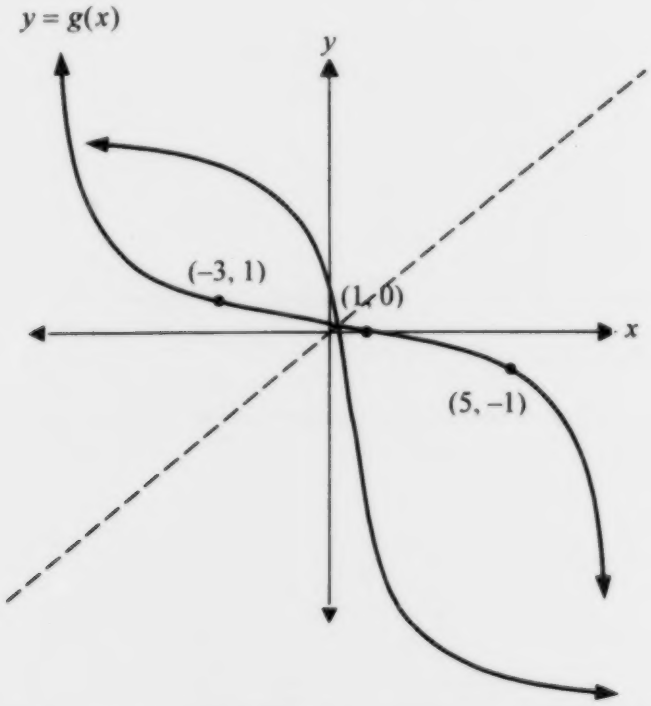
Outcomes: B3, B6

Question

Given the graph of $y = g(x)$:

b) On the axes provided, sketch the graph of $y = g^{-1}(x)$. **1 mark**

Solution



Note(s):

a)

- if a correct explanation of transformations is given in a), but graph is incorrect, give 1 mark
- if student chooses to include coordinates in the solution **deduct** ½ mark for each incorrect coordinate

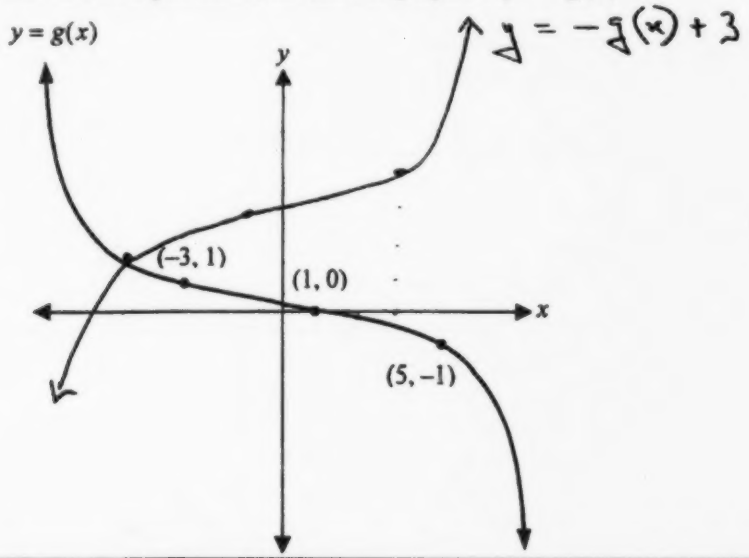
b)

- if graph of reciprocal is given, $y = \dfrac{1}{g(x)}$, award zero marks
- drawing $y = x$ is not essential for full marks

EXEMPLARS

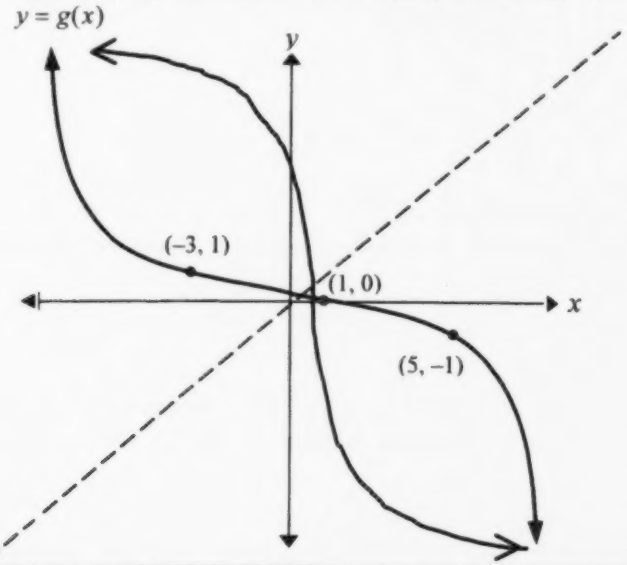
35. Given the graph of $y = g(x)$:

a) On the axes provided, sketch the graph of $y = -g(x) + 3$.



Mark: **1 out of 2**
Note(s):
- deducted 1 mark for reflecting in y-axis
- translated correctly

b) On the axes provided, sketch the graph of $y = g^{-1}(x)$.



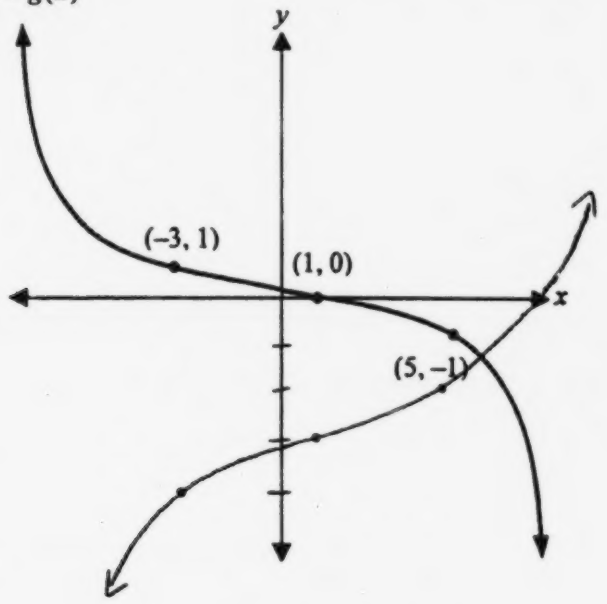
Mark: **½ out of 1**
Note(s):
- ½ mark for correct "general" shape of the inverse
- the curve and its inverse must intersect on the reflection line $y = x$ and the point (1, 0) seems to be reflected into about (0, 4) instead of (0, 1)

EXEMPLAR

35. Given the graph of $y = g(x)$:

a) On the axes provided, sketch the graph of $y = -g(x) + 3$.

$y = g(x)$



Mark:	**1 out of 2**
Note(s):	• deducted 1 mark for incorrect order of transformations • student shifted 3 units up and then reflected graph in x-axis

Question No. 36

Outcomes: C1, C2

Question

36. Find the exact value for $\sec\left(\frac{\pi}{12}\right)$.

4 marks

Solution

Method 1:

$\sec\left(\frac{\pi}{12}\right)$

$\left(\frac{\pi}{12}\right)=\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$ OR $\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$ } 1 mark for combination

$$\cos\left(\frac{\pi}{12}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}$$
$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}$$
$$=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

} 2 marks for correct function values and simplification

deduct ½ mark for each incorrect function value

$\sec\frac{\pi}{12}=\frac{4}{\sqrt{6}+\sqrt{2}}$ OR $\frac{2\sqrt{2}}{\sqrt{3}+1}$ OR $\sqrt{6}-\sqrt{2}$ (after rationalizing denominator) 1 mark for reciprocal

SOME OTHER POSSIBLE COMBINATIONS:

$\frac{\pi}{12}=\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$\frac{\pi}{12}=\left(\frac{5\pi}{6}-\frac{3\pi}{4}\right)$

$\frac{\pi}{12}=\left(\frac{4\pi}{3}-\frac{5\pi}{4}\right)$

$\frac{\pi}{12}=\left(\frac{11\pi}{6}-\frac{7\pi}{4}\right)$

- degrees can be used

Question No. 36

Outcomes: C1, C2

Question

36. Find the exact value for $\sec\left(\frac{\pi}{12}\right)$. 4 marks

Solution

Method 2:

$$\frac{1}{2}\left(\frac{\pi}{6}\right) = \left(\frac{\pi}{12}\right)$$

$$\cos\left(\frac{\pi}{6}\right) = 2\cos^2\left(\frac{\pi}{12}\right) - 1$$ } 1 mark for correct identity

$$\frac{\sqrt{3}}{2} = 2\cos^2\left(\frac{\pi}{12}\right) - 1$$

$$2\cos^2\left(\frac{\pi}{12}\right) = 1 + \frac{\sqrt{3}}{2}$$

$$\cos^2\left(\frac{\pi}{12}\right) = \frac{2+\sqrt{3}}{4}$$ } 1 mark for solving for $\cos^2\left(\frac{\pi}{12}\right)$

since $\frac{\pi}{12}$ is in quad. I $\cos\theta > 0$

$$\cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{2+\sqrt{3}}}{2}$$ } 1 mark for choosing the correct sign

$$\therefore \sec\left(\frac{\pi}{12}\right) = \frac{2}{\sqrt{2+\sqrt{3}}}$$ } 1 mark for reciprocal

Note(s):

- incorrect rationalizing **deduct** ½ mark

EXEMPLAR

36. Find the exact value for $\sec\left(\frac{\pi}{12}\right)$.

$\frac{\pi}{2} = \frac{6\pi}{12}$

$\frac{\pi}{3} = \frac{4\pi}{12}$

$\frac{\pi}{4} = \frac{3\pi}{12}$

$\frac{\pi}{6} = \frac{2\pi}{12}$

$\sec\frac{\pi}{12} = \frac{1}{\cos\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right)} = \frac{1}{\cos\left(\frac{\pi}{12}\right)} =$

$\cos a \cos b - \sin a \sin b$

$\frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$ $\quad \frac{\sqrt{6}+\sqrt{2}}{4} = \frac{4}{\sqrt{6}+\sqrt{2}}$

Mark: **3 out of 4**

Note(s): • Deducted 1 mark for numerous presentation errors:

a) incorrect sequence, should be $\sec\frac{\pi}{12} = \frac{1}{\cos\left(\frac{\pi}{12}\right)} = \frac{1}{\cos\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right)}$

b) $\frac{1}{\cos\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right)} \neq \frac{1}{\cos\left(\frac{3\pi}{12} - \frac{2\pi}{12}\right)}$

c) $\frac{1}{\cos\left(\frac{\pi}{12}\right)} \neq \frac{\sqrt{6}+\sqrt{2}}{4}$

d) $\frac{\sqrt{6}+\sqrt{2}}{4} \neq \frac{4}{\sqrt{6}+\sqrt{2}}$

Question No. 37

Outcomes: C1, C2

Question

37. Prove: $\tan x = \dfrac{1-\cos 2x}{\sin 2x}$

4 marks

Solution

Method 1:

RHS:

$= \dfrac{1-\cos 2x}{\sin 2x}$

$\left(1-2\sin^2 x\right)$ can be used here

$= \dfrac{1-(\cos^2 x-\sin^2 x)}{2\sin x\cos x}$ } 1 mark for substitutions

$= \dfrac{1-\cos^2 x+\sin^2 x}{2\sin x\cos x}$

$= \dfrac{\sin^2 x+\sin^2 x}{2\sin x\cos x}$

$= \dfrac{2\sin^2 x}{2\sin x\cos x}$ } 2 marks (expressing numerator in terms of sin x and simplifying)

$= \dfrac{\sin x}{\cos x}$

$= \tan x$ = LHS } 1 mark

Question No. 37

Outcomes: C1, C2

Question

37. Prove: $\tan x = \dfrac{1-\cos 2x}{\sin 2x}$

4 marks

Solution

Method 2:

RHS:

$$= \frac{1-\cos 2x}{\sin 2x}$$

$$= \frac{1-(2\cos^2 x - 1)}{2\sin x \cos x}$$ } 1 mark for substitution

$$= \frac{2-2\cos^2 x}{2\sin x \cos x}$$

$$= \frac{\cancel{2}\left(1-\cos^2 x\right)}{\cancel{2}\sin x \cos x}$$ } 2 marks (expressing numerator in terms of sin x and simplifying)

$$= \frac{\sin^2 x}{\sin x \cos x}$$

$$= \frac{\sin x}{\cos x}$$ } 1 mark

$$= \tan x = \text{LHS}$$

Note(s):

- the correct format when proving an identity is to take each side separately
- if equality of both sides is indicated throughout the solution **deduct** 1 mark
- **deduct** 1 mark if x is omitted more than once in the proof

EXEMPLAR

37. Prove: $\tan x = \dfrac{1-\cos 2x}{\sin 2x}$

$$RHS = \frac{1-\cos 2x}{\sin 2x}$$

$$= \frac{1-(1-2\sin^2 x)}{2\sin x \cos x}$$

$$= \frac{1-1+2\sin^2 x}{2\sin x \cos x}$$

$$= \frac{2\sin x}{2\cos x} = \frac{\sin x}{\cos x} = \tan x \equiv LHS$$

QED

Mark: **3½ out of 4**

Note(s): • very sloppy presentation on third line. The number 2 seems to be cancelled and reappears in fourth line. $\dfrac{\sin^2 x}{\sin x} = \sin x \neq \sin$ (missing argument)

Question No. 38

Outcomes: G3, G4

Question

38. In a first-year university calculus class, 70% of the students have taken calculus in high school.

The pass rate in university calculus for students who had taken high school calculus is 80%.

The pass rate in university calculus for students who had <u>not</u> taken high school calculus is 40%.

a) If a student took high school calculus, what is the probability of <u>not</u> passing university calculus?

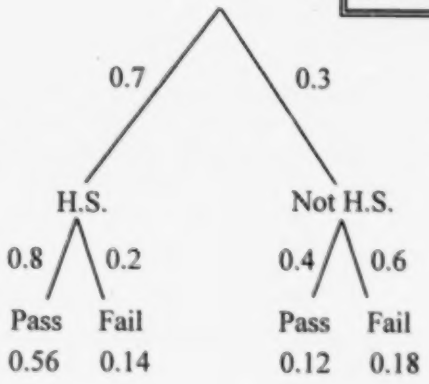
1 mark

b) If a student passes university calculus, what is the probability that he took calculus in high school?

3 marks

Solution

a) Tree diagram (not necessary for part a)



P(Not passing University Calculus/taken H.S. Calculus) $= 1 - 0.8$ (½ mark)
$= 0.2$ (½ mark)

b) P(Pass University Calculus) = (0.7)(0.8) + (0.3)(0.4) = 0.68
or
P(Pass University Calculus) = 0.56 + 0.12 = 0.68
} 1 mark

P(Pass University Calculus and took H.S. Calculus) = 0.56 1 mark

∴ P(took H.S. Calculus / passed University Calculus)

$= \frac{0.56}{0.68}$

$= 0.8235$ or $\frac{14}{17}$
} 1 mark

Note(s):

- in part b), if a correctly labelled tree diagram is provided give 1 mark

EXEMPLARS

38. In a first-year university calculus class, 70% of the students have taken calculus in high school.
The pass rate in university calculus for students who had taken high school calculus is 80%.
The pass rate in university calculus for students who had not taken high school calculus is 40%.

a) If a student took high school calculus, what is the probability of not passing university calculus?

$1 - .8 = .2$

Mark:	**1 out of 1**

b) If a student passes university calculus, what is the probability that he took calculus in high school?

$.70 \times .80 = .56$

$.3 \times .4 = .12$

$.68$

Mark:	**1 out of 3**
Note(s):	• 1 mark for calculating the probability of passing University Calculus—denominator of the probability calculation

Question No. 39

Outcome: D5

Question

39. If $\log_a 2 = x$ and $\log_a 5 = y$, find an algebraic expression for $\log_a\left(\frac{\sqrt[3]{10}}{2}\right)$ in terms of x and y.

3 marks

Solution

$\log_a\left(\frac{\sqrt[3]{10}}{2}\right)$

$\log_a\left(\frac{10^{\frac{1}{3}}}{2}\right)$

$= \log_a 10^{\frac{1}{3}} - \log_a 2$

$= \frac{1}{3}\log_a 10 - \log_a 2$

} 1 mark (½ mark for each theorem)

$= \frac{1}{3}\log_a(2 \times 5) - \log_a 2$

} ½ mark for substitution

$= \frac{1}{3}(\log_a 2 + \log_a 5) - \log_a 2$

} 1 mark for product rule

$= \frac{1}{3}(x+y) - x$ *or* $\frac{1}{3}y - \frac{2}{3}x$

$= \frac{1}{3}x + \frac{1}{3}y - x$ *or* $\frac{y-2x}{3}$

} ½ mark for substitution

EXEMPLAR

39. If $\log_a 2 = x$ and $\log_a 5 = y$, find an algebraic expression for $\log_a\left(\frac{\sqrt[3]{10}}{2}\right)$ in terms of x and y.

$$\log_a\left(\frac{\sqrt[3]{10}}{2}\right) = \log_a \sqrt[3]{10} - \log_a 2$$

$$= \frac{1}{3}\log_a 10 - \log_a 2$$

$$= \frac{1}{3}\left(\log_a 5 + \log_a 5\right) - \log_a 2$$

$$= \frac{1}{3}(2y) - x$$

$$= \frac{2y}{3} - x$$

Mark: **2 out of 3**

Note(s): • deducted 1 mark for conceptual error $\log_a 10 \neq \log_a 5 + \log_a 5$

Question No. 40

Outcomes: D3, D7

Question

40. Solve for x algebraically.
Show all steps.

$\ln\left(e^{\sqrt{x}}\right) = 8$

2 marks

Solution

$\ln\left(e^{\sqrt{x}}\right) = 8$

$\sqrt{x}\ \ln e = 8$
$\sqrt{x}\ (1) = 8$
} ½ mark for log theorem
½ mark for $\ln e = 1$

$\sqrt{x} = 8$
$x = 64$
} 1 mark for solving for x

Note(s):

- give ½ mark if only given $\sqrt{x}\ln e = 8$

EXEMPLAR

40. Solve for x algebraically.
Show all steps.

$$\ln\left(e^{\sqrt{x}}\right) = 8$$

$\sqrt{x} = 8$

$x = \sqrt{8}$ or $x = 2.8284$

Mark:	**1 out of 2**
Note(s):	• 1 mark for treating $\ln x$ and e^x as inverse functions

Question No. 41

Outcome: D8

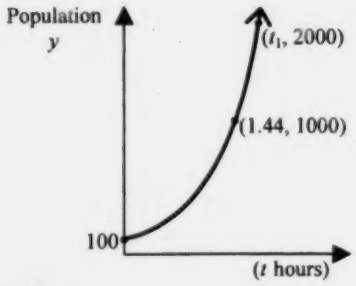
Question

41. The population growth of a bacterial culture follows an exponential curve $y = 100e^{kt}$ as shown in the diagram.

Find the value of t_1, the time that the population reaches 2000.

3 marks

Round your final answer to two decimal places.



Solution

Method 1:

$y = 100e^{kt}$

$1000 = 100e^{k(1.44)}$ ½ mark for numerical substitution from graph

$10 = e^{1.44k}$

$\ln 10 = 1.44k$ } 1 mark for solving for k

$k = 1.599$

$2000 = 100e^{1.599t_1}$ ½ mark for substitution of k value

$20 = e^{1.599t_1}$

$\ln 20 = 1.599t_1$ } 1 mark for solving for t_1

$t_1 = 1.87$ hours

Method 2:

$10 = e^{1.44k}$ ½ mark for substitution

$2000 = 100e^{kt_1}$ ½ mark for substituting the second point

$20 = e^{kt_1}$

$20 = \left(e^{1.44k}\right)^{\frac{t_1}{1.44}}$ } 1 mark for substituting 10 for $e^{1.44k}$

$20 = 10^{\frac{t_1}{1.44}}$

$\log 20 = \frac{t_1}{1.44}$

$t_1 = 1.44 \log 20$ } 1 mark for solving for t_1

$t_1 = 1.87$ hours

Question No. 41

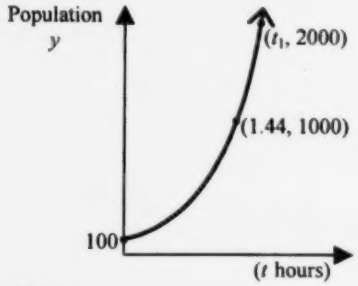
Outcome: D8

Question

41. The population growth of a bacterial culture follows an exponential curve $y = 100e^{kt}$ as shown in the diagram.

    Find the value of t_1, the time that the population reaches 2000.

    Round your final answer to two decimal places. **3 marks**



Solution

Method 3:

Using graphing calculator.

$y_1 = 100e^{1.44x}$ $y_2 = 1000$	½ mark for both equations
Calculate intersect. $x = 1.599$	½ mark for identifying $x = 1.599$ as x-coordinate of **intersection point** ½ mark for value of x
$y_1 = 100e^{1.599x}$ $y_2 = 2000$	½ mark for both equations
Calculate intersect	½ mark for identifying $x = 1.87$ as an **intersection point**
$t_1 = 1.87$ hours	½ mark for value of t_1

Note(s):

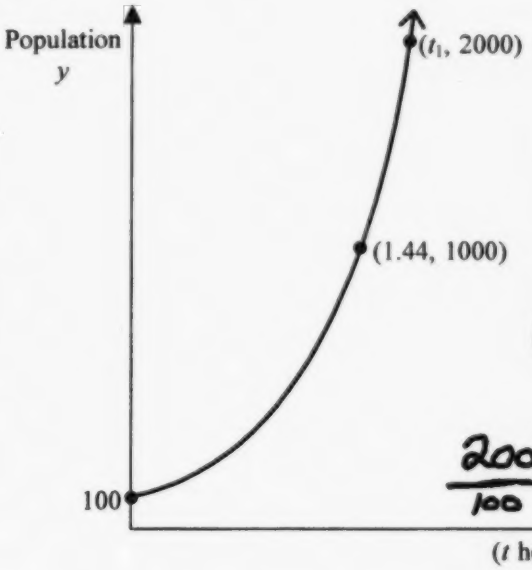
- **deduct** ½ mark if 1.87 hours is written as 1 hour and 87 minutes

EXEMPLAR

41. The population growth of a bacterial culture follows an exponential curve $y = 100e^{kt}$ as shown in the diagram.

    Find the value of t_1, the time that the population reaches 2000.

    Round your final answer to two decimal places.



$\frac{1000}{100} = \frac{100e^{k(1.44)}}{100}$

$\frac{10}{1.44} = \frac{k(1.44)}{1.44}$

$6.9\bar{4} = k$

$\frac{2000}{100} = \frac{100e^{(6.9\bar{4})T}}{100}$

$\frac{20}{6.9\bar{4}} = \frac{(1.9\bar{4})(T)}{6.9\bar{4}}$

$2.88 = T$

Mark:	**1 out of 3**
Note(s):	• ½ mark for numerical substitution from graph
	• ½ mark for substitution of k value

Question No. 42

Outcome: E4

Question

42. Find a simplified expression for the fourth term in the expansion of $\left(x^2 - \frac{2}{x}\right)^{10}$. **3 marks**

Solution

The 4^{th} term is:

$${}_{10}C_3(x^2)^7\left(\frac{-2}{x}\right)^3$$ } 1 mark for correct setup

$$= 120(x^{14})\left(\frac{-8}{x^3}\right)$$ } 1 mark for simplification

$$= -960x^{11}$$ } 1 mark < ½ mark for numerical value, – 960; ½ mark for x^{11}

Notes:

- may use ${}_{10}C_7$ for coefficient
- if student uses ${}_{10}C_4$ and follows through correctly, give 2 marks

$${}_{10}C_4\left(x^2\right)^6\left(\frac{-2}{x}\right)^4 = 3360x^8$$

EXEMPLAR

42. Find a simplified expression for the fourth term in the expansion of $\left(x^2 - \frac{2}{x}\right)^{10}$.

$10C_3 = 120\left(x^{(2)^7} - \frac{2}{x}^3\right)$

$120\left(x^{14} - \frac{8}{x^3}\right)$

$-840x^{11}$

Mark: 1½ out of 3

Note(s):
- deducted ½ mark for equating ${}_{10}C_3$ to the term
- deducted ½ mark for writing $\left(x^2\right)^7\left(\frac{-2}{x}\right)^3$ as $\left(x^2\right)^7 - \frac{2^3}{x}$
- deducted ½ mark for arithmetic error, $120(8) \neq 840$

Question No. 43

Outcome: E2

Question

43. In how many ways can 7 people be seated at a round table if 2 people refuse to sit next to each other?

2 marks

Solution

Method 1:

There are 6! = 720 total seating arrangements at the table.

If the 2 people sit together, there would be a total of $2 \times 5! = 240$ arrangements

$\therefore$ If they refuse to sit together, there are

$6! - 2(5!)$ } 1½ marks

$= 720 - 240$

$= 480$ arrangements

- ½ mark for 6!
- ½ mark for 2(5!)
- ½ mark for subtraction
- ½ mark for answer

Method 2:

½ mark ½ mark ½ mark ½ mark

$1 \times 4 \times 5! = 480$

- 1: First person, of the pair, forms a reference
- 4: Second person, of the pair, has 4 choices of seats
- 5!: The other 5 people can be arranged in the remaining seats in 5! ways.

EXEMPLAR

43. In how many ways can 7 people be seated at a round table if 2 people refuse to sit next to each other?

$(5-1)! \cdot 2! = 48$

$(7-1)! = 720$

$720 - 48 = \underline{672}$

Mark: **1½ out of 2**
Note(s): • deducted ½ mark for writing (5–1)!2! instead of (6–1)!2!

Part 2—Short Answer

Question No. 44

Outcome: A3

Question

44. What is the value of $\cos^2(30°) + \sin^2(60°)$?

1 mark

Answer

$\cos^2(30^\circ) + \sin^2(60^\circ)$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$\frac{3}{4} + \frac{3}{4}$

$\frac{6}{4} = \frac{3}{2}$

Note(s):

- give ½ mark for one correct substitution and correct carry error
- give ½ mark for correct substitution but incorrect final answer

Question No. 45

Outcome: A1

Question

45. Express 13° in radian measure.
State your answer as an exact value.

1 mark

Answer

$$\frac{13^\circ}{180^\circ} = \frac{R}{\pi} \qquad \text{OR} \qquad 13^\circ \times \frac{\pi}{180^\circ} = \frac{13\pi}{180}$$

$$R = \frac{13\pi}{180}$$

Note(s):

- give ½ mark if correct proportion is given but incorrect final answer
- **deduct** ½ mark if one degree symbol is left on final answer
- $\frac{13^\circ\pi}{180^\circ}$ is correct

Question No. 46

Outcome: B5

Question

46. What is the range of the function $y = |2\cos x|$?

1 mark

Answer

Range: $[0,2]$ OR $0 \le y \le 2$

Note(s):

- give ½ mark for $(0,2)$ OR $0 < y < 2$ OR $(0,2]$ OR $0 < y \le 2$ OR $[0,2)$ OR $0 \le y < 2$
- any other incorrect answer give 0 marks

Question No. 47

Outcome: B3

Question

47. If the x-intercept of $y = f(x)$ is 4, what is the value of the x-intercept for $y = 2f(-x)$?

1 mark

Answer

x intercept is -4

Note(s):

- give ½ mark for $(-4, 0)$

Question No. 48

Outcome: C2

Question

48. What is the maximum value of $3(\cos^2 x - \sin^2 x)$?

1 mark

Answer

$$3(\cos^2 x - \sin^2 x)$$
$$= 3(\cos 2x)$$

Maximum value is 3

Note(s):

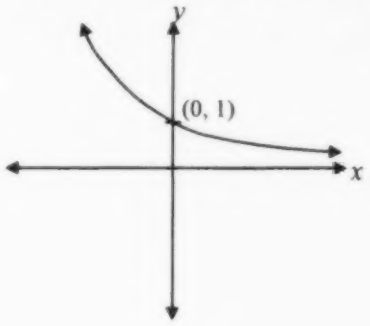
- give ½ mark if student recognizes $\cos^2 x - \sin^2 x$ as being $\cos 2x$

Question No. 49

Outcome: D1

Question

49. The graph of the exponential function $y = a^x$ is shown below. State a possible value for a.



1 mark

Answer

- accept any value of a in the interval (0, 1)

  example: $a = \frac{1}{2}$

Note(s):

- if student gives interval $0 < a < 1$ OR $(0,1)$ then give 1 mark
- give ½ mark for $(0,1]$ OR $0 < a \leq 1$ OR $[0,1)$ OR $0 \leq a < 1$

Question No. 50

Outcome: D2

Question

50. Solve for x:

$125 = 5^{2x-1}$

1 mark

Answer

$5^3 = 5^{2x-1}$

$2x - 1 = 3$

$2x = 4$

$x = 2$

Note(s):

- give ½ mark if correct exponents are equated but answer is incorrect

Question No. 51

Outcome: E1

Question

51. There are 6 finalists in the 100-meter dash at the Snow Lake Track Meet.
In how many different ways can first and second place medals be awarded?
Assume that there are no ties.

State your answer as a whole number.

1 mark

Answer

$6 \times 5 = 30$

Note(s):

- give ½ marks for :

${}_6C_2 \times 2$ OR 6×5 OR ${}_6P_2$ OR an equivalent expression

Question No. 52

Outcome: F1

Question

52. What type of conic is represented by the following equation?

1 mark

$$x^2 + 4x - 2y + 12 = 0$$

Answer

parabola

Question No. 53

Outcome: G2

Question

53. A bag contains x red balls and y yellow balls.
A ball is drawn at random from the bag.
Find an algebraic expression to represent the probability that the ball is red. **1 mark**

Answer

$$\text{Probability (red)} = \frac{x}{x+y}$$

Note(s):

- if the final answer is incorrect, $\left(\text{e.g., } P(\text{red}) = 1 + \frac{1}{y}\right)$
but student initially stated $\text{Probability (red)} = \frac{x}{x+y}$ give ½ mark

Part 2—Long Answer

Question No. 54

Outcome: B4

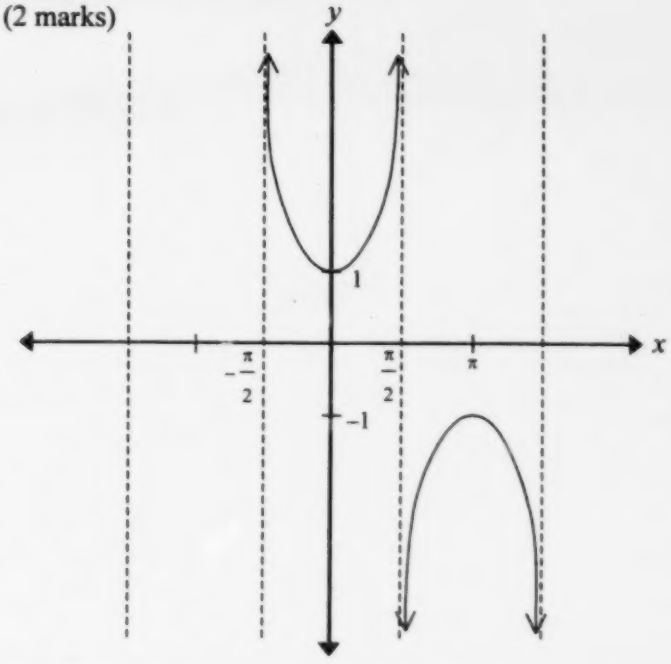
Question

54. Sketch a clearly labelled graph of $y = \sec x$ showing at least one period.
State the range of this function.

3 marks

Solution

Graph: (2 marks)



Range: (1 mark)

$(-\infty, -1] \cup [1, \infty)$

OR

$\{y \mid y \geq 1 \text{ or } y \leq -1\}$

OR

$\{\ y \mid y \notin (-1, 1)\ \}$

Note(s):

- give 2 marks for accurate graph and at least one full period
- give 1 mark for an accurate graph, showing at least one full period, of $y = \csc x$
- **deduct** ½ mark for incorrect y-intercept
- **deduct** ½ mark for no asymptotes
- **deduct** ½ mark for only half a period graphed
- **deduct** ½ mark for each axis not labelled with scale
- if $y = \sec x$ and $y = \cos x$ are both drawn and $y = \sec x$ is not clearly identified **deduct** ½ mark

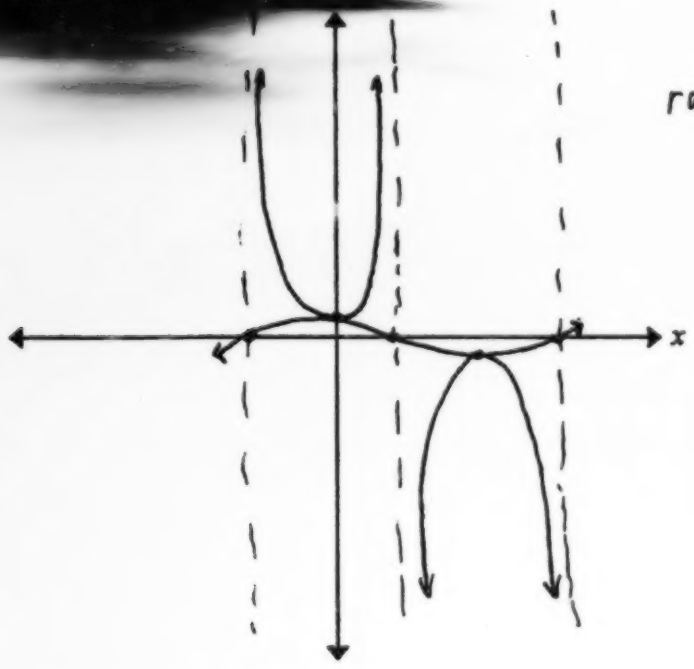
Range Note(s):

- **deduct** ½ mark for $(-\infty, -1) \cup (1, \infty)$

EXEMPLAR

54. Sketch a clearly labelled graph of $y = \sec x$ showing at least one period.

[illegible] of this function.



range: $y \neq 0$

Mark:	**½ out of 3**
Note(s):	• deducted 1 mark (deducted ½ mark for no identification of the scale on each of the axis) • deducted ½ mark because $y = \sec x$ is not clearly identified • deducted 1 mark for incorrect range

Question No. 55

Outcomes: F1, B1

Question

55. Compare the graphs of the relations $x^2 = y$ and $(x-2)^2 = y+4$.
(State one similarity and one difference.)

2 marks

Solution

Similarities (1 mark for one similarity)

- Both are parabolas
- Both have vertices
- Both graphs open up

Difference (1 mark for one difference)

- $y = x^2$ has vertex at (0, 0) and $y + 4 = (x - 2)^2$ has vertex at (2, –4)
- $y = x^2$ is symmetrical about the y-axis, $y + 4 = (x-2)^2$ is not
- second graph is original (or first) graph translated 4 units down and 2 units right

Note(s):

- any similarity or difference must be given with respect to the graph
- if only a correctly labelled graph of both relations is drawn, give one mark

EXEMPLARS

55. Compare the graphs of the relations $x^2 = y$ and $(x-2)^2 = y+4$.
(State one similarity and one difference.)

they are both parabolas, the second has been shifted 4 up and slid 2 units right.

Mark:	**1½ out of 2**
Note(s):	• deducted ½ mark for incorrect vertical shift (should be 4 down not 4 up)

55. Compare the graphs of the relations $x^2 = y$ and $(x-2)^2 = y+4$.
(State one similarity and one difference.)

similarity is that it is still the shape of x^2
difference is that it is down 4

Mark:	**1 out of 2**
Note(s):	• deducted ½ mark for unclear similarity (x^2 is not a shape) • deducted ½ mark for difference because there is no indication of which graph has been moved down 4 units.

Question No. 56

Outcomes: F2, F3

Question

56. a) Write the equation $9x^2 - 18x + 4y^2 + 16y - 11 = 0$ in standard form. | 3 marks |

b) Sketch a clearly labelled graph of the above equation. | 1 mark |

Solution

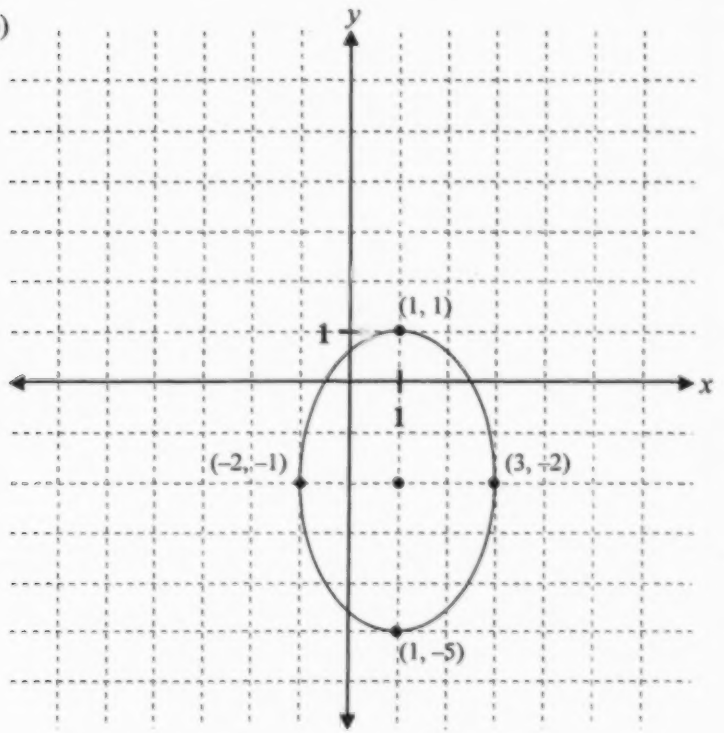
a) $9x^2 - 18x + 4y^2 + 16y = 11$

$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9 + 16$ } 2 marks→ 1 mark for each completed square

$9(x-1)^2 + 4(y+2)^2 = 36$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$ } 1 mark for standard form

b)



½ mark for elliptical shape

½ mark for all correct vertices (coordinates of vertices need not be stated.)

Note(s):

b)

- if elliptical graph is consistent with equation in part a) give 1 mark

EXEMPLAR

56. a) Write the equation $9x^2 - 18x + 4y^2 + 16y - 11 = 0$ in standard form.

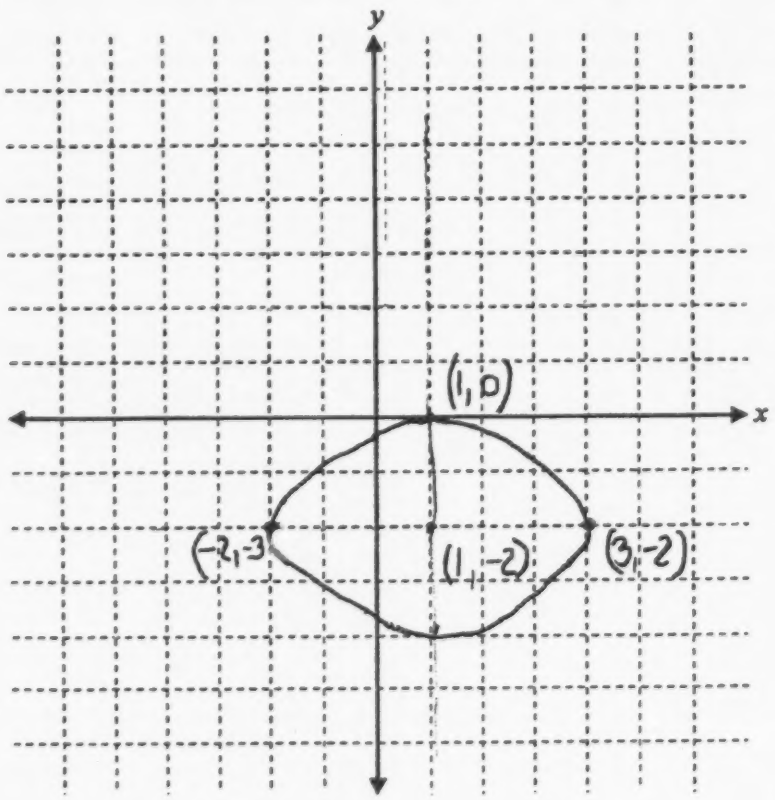
$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 0 + 11 + 9 + 16$

$9(x-1)^2 + 4(y+2)^2 = 36 \mid :36$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$

Mark: 3 out of 3

b) Sketch a clearly labelled graph of the above equation.



Mark: ½ out of 1
Note(s): • deduct ½ mark for incorrect vertices

Question No. 57

Outcome: G2

Question

57. Jack and Jill are students in a class of 20.

A committee of 5 students is to be selected from the class.

Find the probability that both Jack and Jill will be selected for the committee.

Express your answer as a number in its simplest form.

3 marks

Solution

Method 1:

There are ${}_{20}C_5$ total possible committees.

If Jack and Jill are selected, there are a total of ${}_{18}C_3$ possible committees.

$\therefore P(\text{Jack and Jill}) = \dfrac{{}_{18}C_3}{{}_{20}C_5}$ *OR* $\dfrac{{}_{18}C_3 \bullet {}_2C_2}{{}_{20}C_5}$ 1 mark for setup

$$= \frac{\dfrac{18!}{15!\,3!}}{\dfrac{20!}{15!\,5!}}$$ } 1 mark for factorials

$$= \frac{18!}{15!3!} \bullet \frac{15!5!}{20!}$$

$$= \frac{18! \bullet 5 \bullet 4 \bullet 3!}{3! \bullet 20 \bullet 19 \bullet 18!}$$ } ½ mark for reducing/cancelling

$$= \frac{5 \bullet 4}{20 \bullet 19}$$

$$= \frac{1}{19}$$ } ½ mark for simplified answer

Question No. 57

Outcome: G2

Question

57. Jack and Jill are students in a class of 20.

A committee of 5 students is to be selected from the class.

Find the probability that both Jack and Jill will be selected for the committee.

Express your answer as a number in its simplest form.

3 marks

Solution

Method 2:

P(Jack and Jill) = 1 – P(neither Jack nor Jill OR Jill, but not Jack OR Jack, but not Jill) 1 mark for method

$$= 1 - \left(\frac{{}_{18}C_5}{{}_{20}C_5} + \frac{{}_1C_1 \bullet {}_{18}C_4}{{}_{20}C_5} + \frac{{}_1C_1 \bullet {}_{18}C_4}{{}_{20}C_5} \right)$$

1½ marks (½ mark for each correct probability)

$$= 1 - \left(\frac{21}{38} + \frac{15}{76} + \frac{15}{76} \right)$$

$$= 1 - \left(\frac{72}{76} \right) = 1 - \frac{18}{19} = \frac{1}{19}$$

½ mark for simplified answer

Note(s):

- if initial setup has one incorrect combination (e.g., ${}_{18}C_3$ or ${}_{20}C_5$ is incorrect) but subsequent calculations are correct, **deduct** only 1 mark
- if both initial setup combinations are incorrect but subsequent calculations are correct, give maximum 1 mark
- If 1 – P(not Jack and Jill)

$$= 1 - \left(\frac{{}_{18}C_5}{{}_{20}C_5} \right) = 1 - \frac{21}{38} = \frac{17}{38}$$ (maximum 1 mark)

EXEMPLAR

57. Jack and Jill are students in a class of 20.

A committee of 5 students is to be selected from the class.

$P(J+J \text{ in comm}) = \frac{5}{68}$

Find the probability that both Jack and Jill will be selected for the committee.

Express your answer as a number in its simplest form.

$_{20}C_5$ = total $\quad \frac{20!}{5!(20-5)!}$

rest of class $\quad J+J$

$\frac{{}_{20}C_3 \cdot {}_2C_2}{{}_{20}C_5 \text{(total)}} = \frac{{}_{20}C_3}{{}_{20}C_5} = \frac{\frac{20!}{3!(20-3)!}}{\frac{20!}{5!(20-5)!}}$

$\frac{5!\,15!}{3!\,17!} = \frac{5 \cdot 4}{1 \cdot 2} = \frac{5}{68}$

$\frac{20!}{3!(20-3)!} \cdot \frac{5!(20-5)!}{20!} = \frac{5!(20-5)!}{3!(20-3)!} = \frac{5!\,15!}{3!\,17!}$

Mark:	**2 out of 3**
Note(s):	• deducted 1 mark for setup (${}_{20}C_3$ should be ${}_{18}C_3$)

Question No. 58

Outcomes: C1, A4

Question

58. Find the exact value(s) of x:

$\sin^2 x - 3\cos x = 3$, where $0 \le x < 2\pi$.

4 marks

Solution

$\sin^2 x - 3\cos x - 3 = 0$		
$1 - \cos^2 x - 3\cos x - 3 = 0$		1 mark for identity
$\cos^2 x + 3\cos x + 2 = 0$		
$(\cos x + 2)(\cos x + 1) = 0$		1 mark for factoring
$\cos x = -2$	$\cos x = -1$	1 mark total—½ mark for each value of $\cos x$
No solution	$x = \pi$	1 mark for consistent answer

Note(s):

- stating "No solution" is optional
- if $x = 180^\circ$ is given instead of $x = \pi$ **deduct** ½ mark
- any solution for $\cos x = -2$ **deduct** 1 mark

EXEMPLAR

58. Find the exact value(s) of x:

$\sin^2 x - 3\cos x = 3$, where $0 \le x < 2\pi$.

$(1-\cos^2 x) - 3\cos x = 3$

$-\cos^2 x - 3\cos x - 2 = 0$

$-x^2 - 3x - 2 = 0$

$x^2 + 3x + 2$

$(x+2)(x+1)$

$x = -2, -1$

$\cos = -2, -1$

$\cos = -1$

$\cos = \pi$

Mark:	**2 out of 4**
Note(s):	• deducted ½ mark for incorrect substitution of x for cos x • deducted ½ mark for changing an equation to an expression • deducted 1 mark for incorrect solution for x (cos is NOT x)

Question No. 59

Outcome: H1

Question

59. Find t_8 in a geometric sequence where $t_1 = 16$ and $t_4 = 2$. **2 marks**

Solution

Method 1:

$t_4 = t_1 r^3$

$2 = 16r^3$

$r^3 = \frac{1}{8}$

$r = \frac{1}{2}$ } 1 mark for stating the value of r

$t_8 = 16 \bullet r^7$

$= 16 \bullet \left(\frac{1}{2}\right)^7$ } ½ mark for substitution

$= \frac{2^4}{2^7} = \frac{1}{2^3}$

$= \frac{1}{8}$ } ½ mark for consistent answer

Method 2:

Sequence is :

$16, 8, 4, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

t_1 t_4 t_8

} 1 mark for identifying sequence
1 mark for correct value of t_8

EXEMPLAR

59. Find t_8 in a geometric sequence where $t_1 = 16$ and $t_4 = 2$.

$r = \frac{1}{2}$

$t_8 = 16\left(\frac{1}{2}\right)^7$

$t_8 = 16\left(\frac{7}{128}\right)$

$t_8 = \frac{112}{128}$

$t_8 = \frac{7}{8}$

16
7
112

Mark:	**1½ out of 2**
Note(s):	• deducted ½ mark for arithmetic error. $\left(\frac{1}{2}\right)^7 \neq \frac{7}{128}$

Question No. 60

Outcome: H2

Question

60. Evaluate $\sum_{i=2}^{4}(i+1)^2$.

2 marks

Solution

$=(2+1)^2+(3+1)^2+(4+1)^2$ 1 mark for terms

$=3^2+4^2+5^2$

$=9+16+25$

$=50$ 1 mark for final answer

Note(s):

- **deduct** 1 mark for incorrect number of terms
  e.g., $(1+1)^2+(2+1)^2+(3+1)^2+(4+1)^2$

EXEMPLAR

60. Evaluate $\sum_{i=2}^{4}(i+1)^2$.

$$S_7 = \frac{t_1(1-r^n)}{1-r}$$

$t_1 = 3^2 = 9$

$t_2 = 4^2 = 16$

$t_3 = 5^2 = 25$

$r = \frac{16}{9} \neq \frac{25}{16}$ ∴ this is not a geo sequence and cannot be done

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$-1\frac{7}{9} \neq 1\frac{9}{16}$

Mark:	**1 out of 2**
Note(s):	• 1 mark given for finding the correct values and correct number of terms